Robust Topological Construction of All-Hexahedral Boundary Layer Meshes

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Context: boundary layer meshing

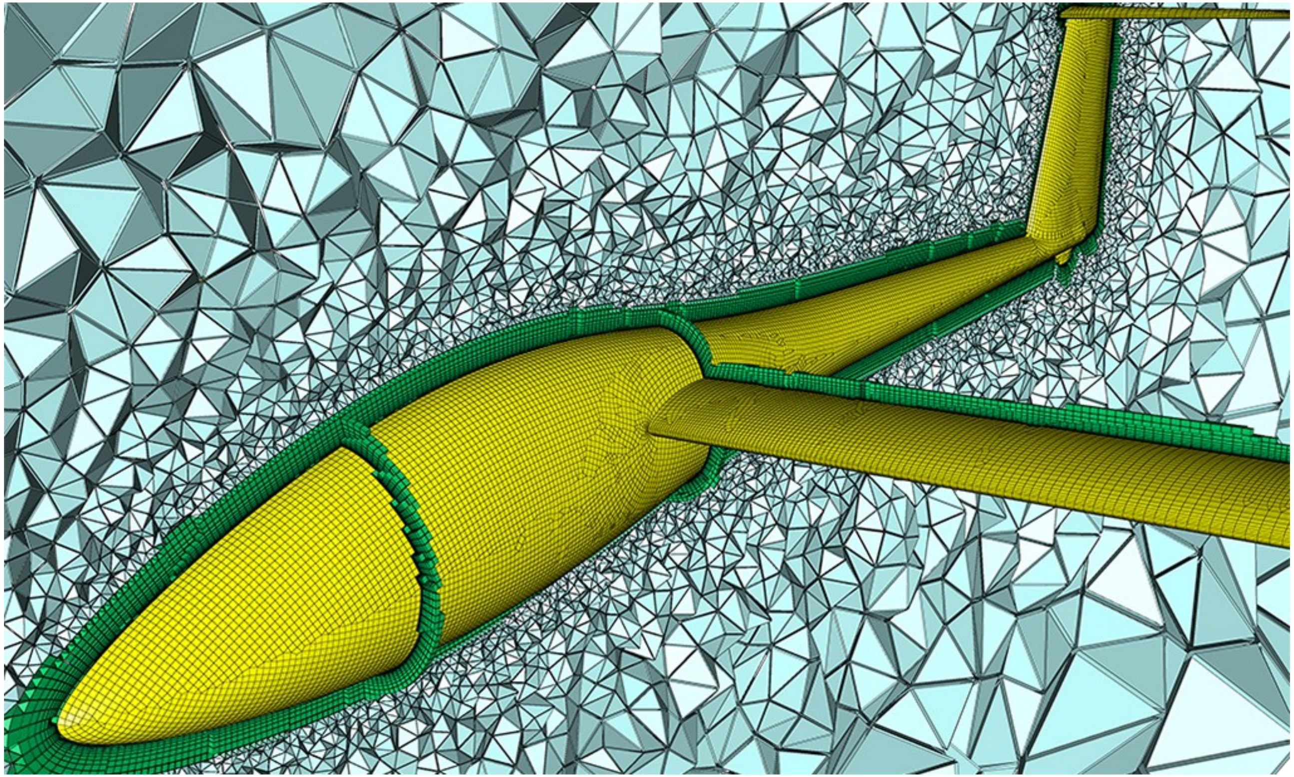


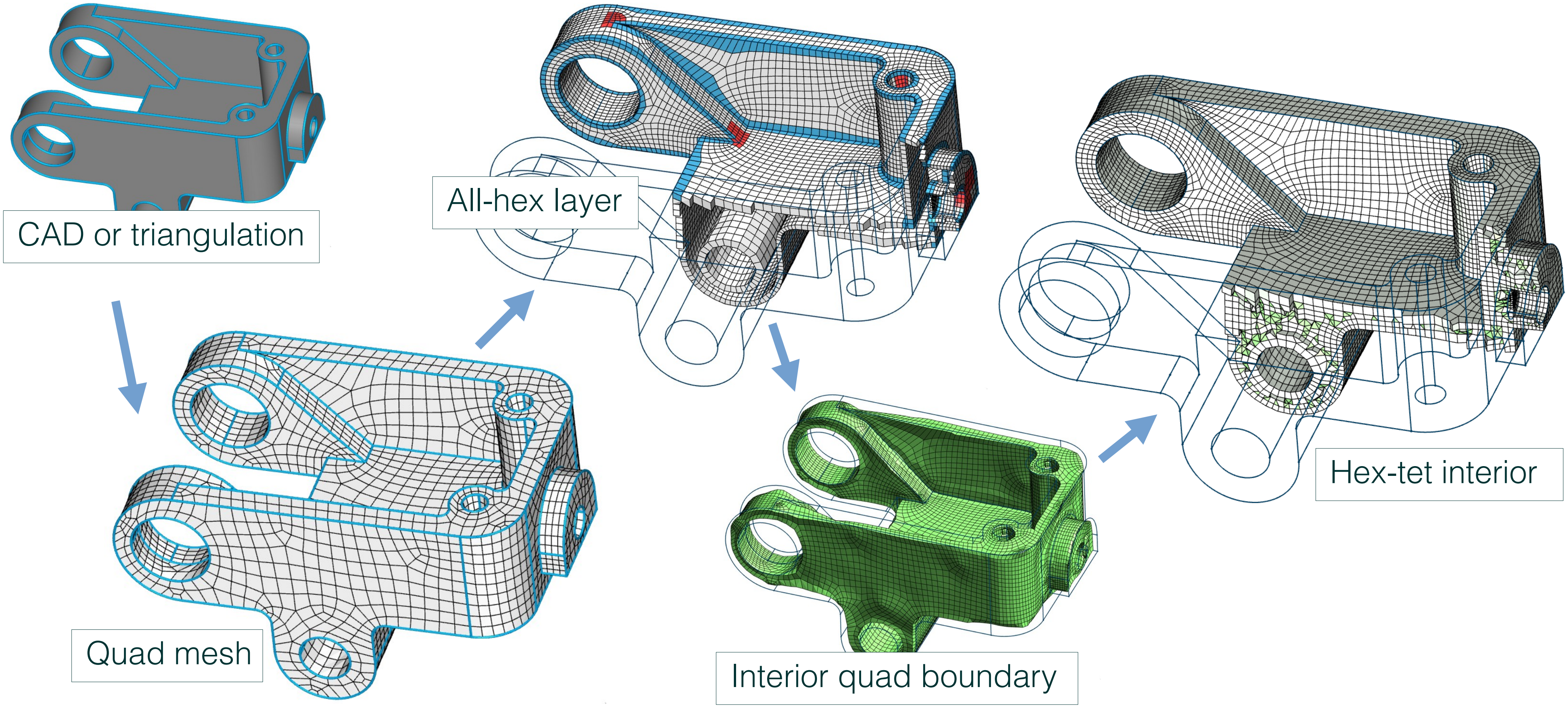
Image from Cadence/Pointwise blog
“Reduce CFD meshing time and improve accuracy with hexahedral boundary layer meshes”

Boundary layer meshes are often critical to capture the physics (e.g. CFD)

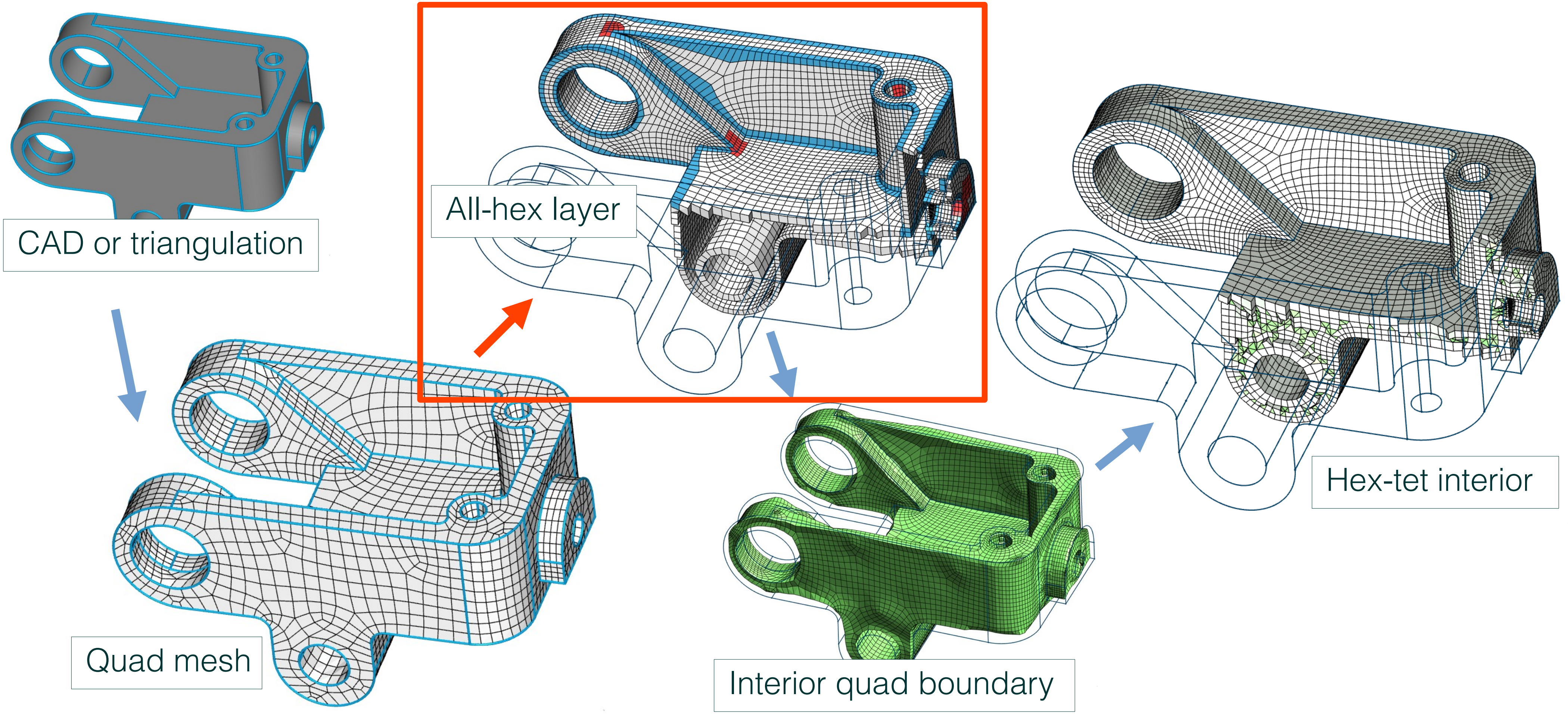
Standard surface-to-volume approach:
- extrusion of surface mesh
- unstructured mesh far from bdr

Quad mesh → hex bdr layers

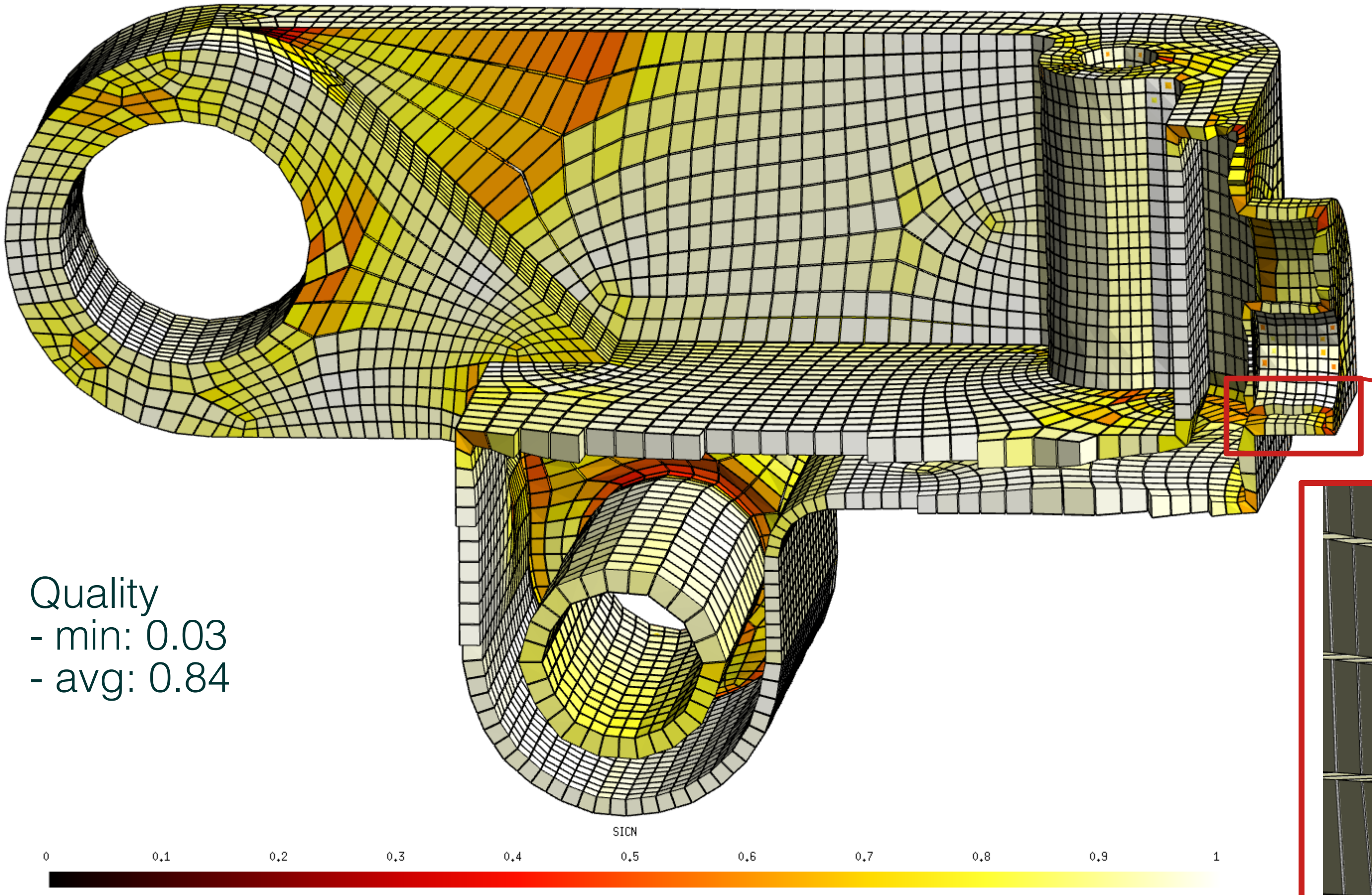
Context: all-hex layer for hex-dominant meshing pipeline

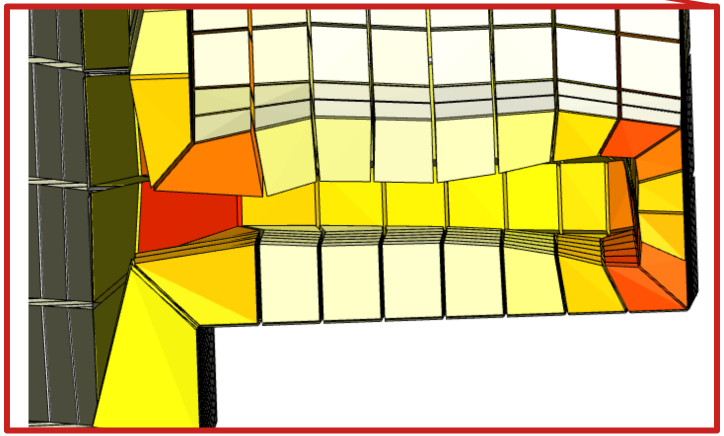


Context: all-hex layer for hex-dominant meshing pipeline



Surface quad mesh extrusion





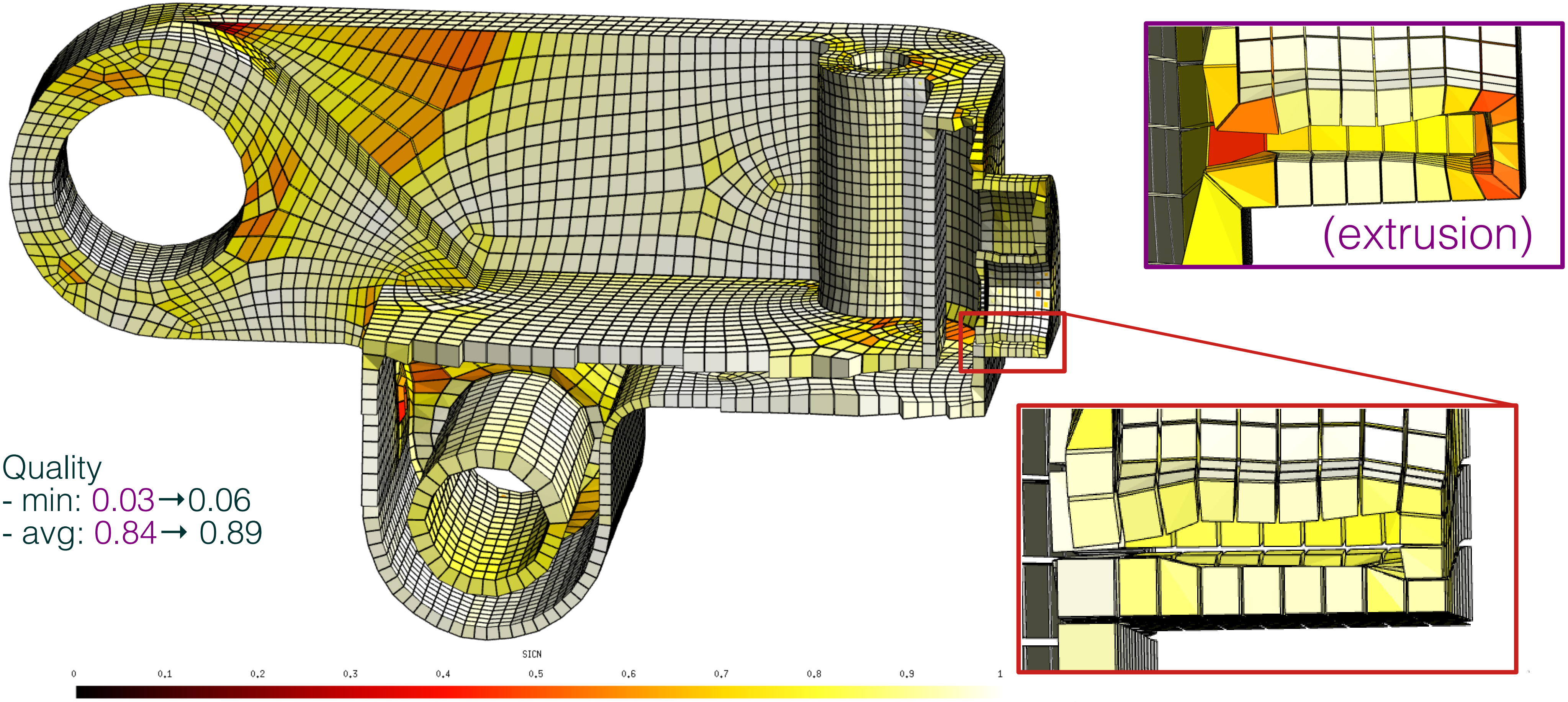
Quality
- min: 0.03
- avg: 0.84

Topology is trivial:
- duplicate the quad mesh

Low-quality hexahedra
on ridges/corners

Curves always split in two

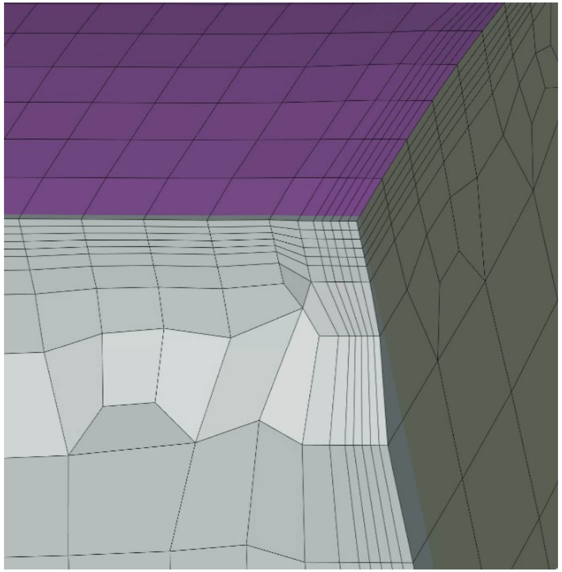
All-hex layer with better boundary valences



Quality
- min: 0.03→0.06
- avg: 0.84→ 0.89

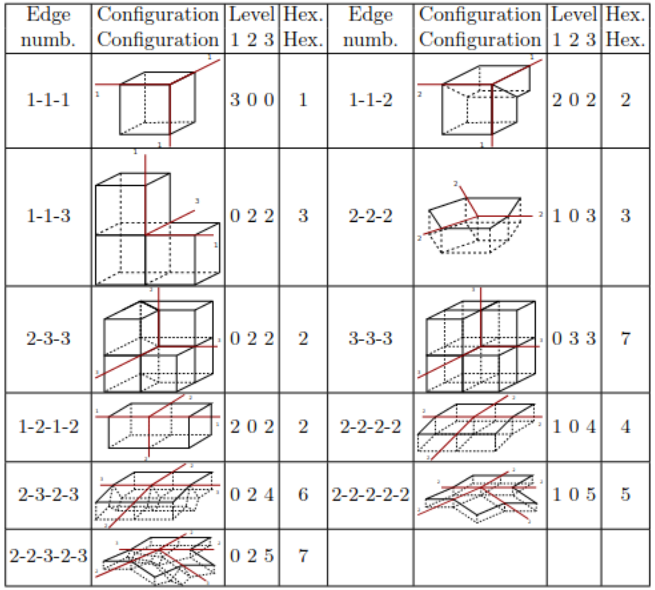
Existing solutions to improve hex boundary layers

Boundary layer cross-imprinting for adjacent surfaces



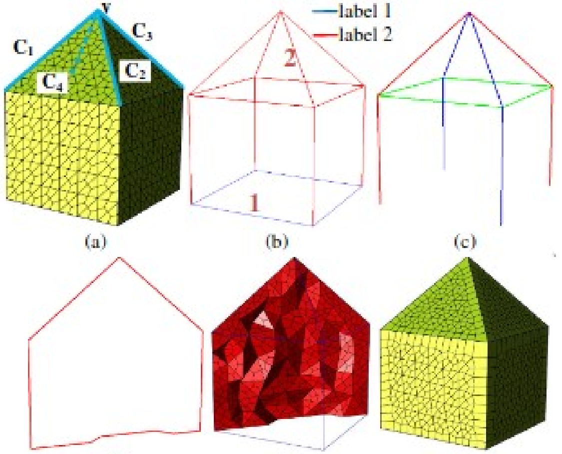
(Maréchal 2016)

A Constraint-Based System for Hexahedral Mesh Transformation



Edge numb.	Configuration Configuration	Level 1 2 3	Hex. Hex.	Edge numb.	Configuration Configuration	Level 1 2 3	Hex. Hex.
1-1-1		3 0 0	1	1-1-2		2 0 2	2
1-1-3		0 2 2	3	2-2-2		1 0 3	3
2-3-3		0 2 2	2	3-3-3		0 3 3	7
1-2-1-2		2 0 2	2	2-2-2-2		1 0 4	4
2-3-2-3		0 2 4	6	2-2-2-2-2		1 0 5	5
2-2-3-2-3		0 2 5	7				

(Ledoux et al. 2013)



$$
\begin{aligned}
&-1 \le \alpha_1 - 2m_1 + \alpha_2 - 2m_2 + \alpha_3 - 2m_3 - 3m_4 \le 0\\
&0 \le \alpha_1 - 2m_1 \le 1\\
&0 \le \alpha_2 - 2m_2 \le 1\\
&0 \le \alpha_3 - 2m_3 \le 1\\
&\alpha_1 - \alpha_2 - 2m_5 + \alpha_2 - \alpha_3 - 2m_6 + \alpha_3 - \alpha_1 - 2m_7 \le 2\\
&0 \le \alpha_1 - \alpha_2 - 2m_5 \le 1\\
&0 \le \alpha_2 - \alpha_3 - 2m_6 \le 1\\
&0 \le \alpha_3 - \alpha_1 - 2m_7 \le 1\\
&m_1, m_2, m_3, m_4, m_5, m_6, m_7 \in \mathbb{Z}
\end{aligned}
$$

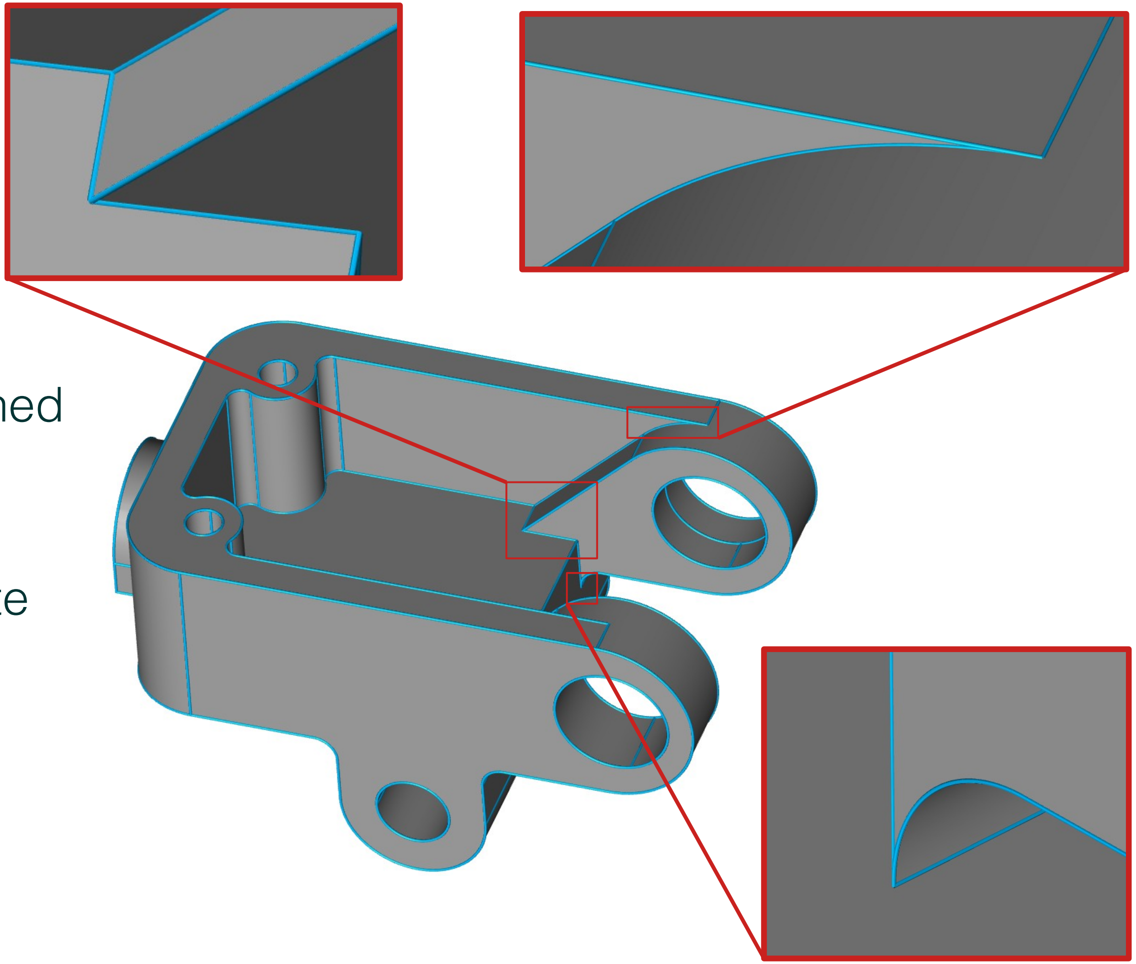
(Wang et al. 2020)

- Limited number of supported configurations
- Global constraint propagation

How to find good all-hex layers ?

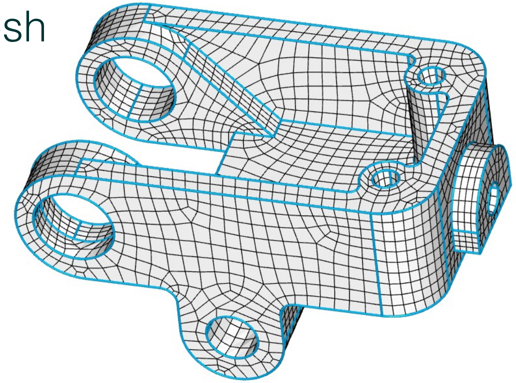
The general case is complicated:

- feature curves junctions are not cross-aligned
- hex mesh topology is (very) constrained
- hex mesh topological constraints propagate

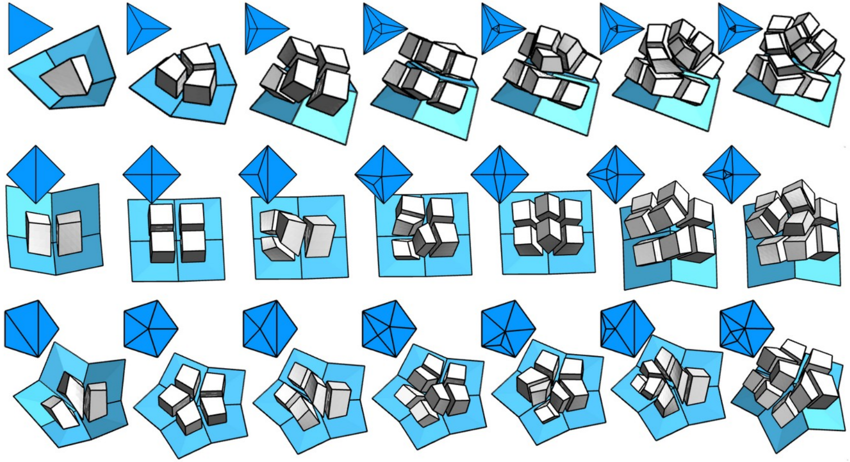


New generic approach to build all-hex boundary layer topology

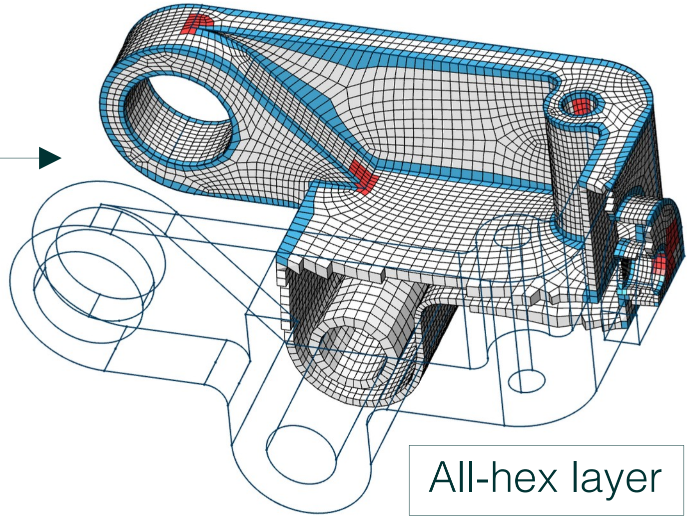
Quad mesh



Consider all possible local boundary hex configurations (one per bdr vertex)



Solver to find the best combination



All-hex layer

Optimization problem: closest solution to ideal edge hex valences

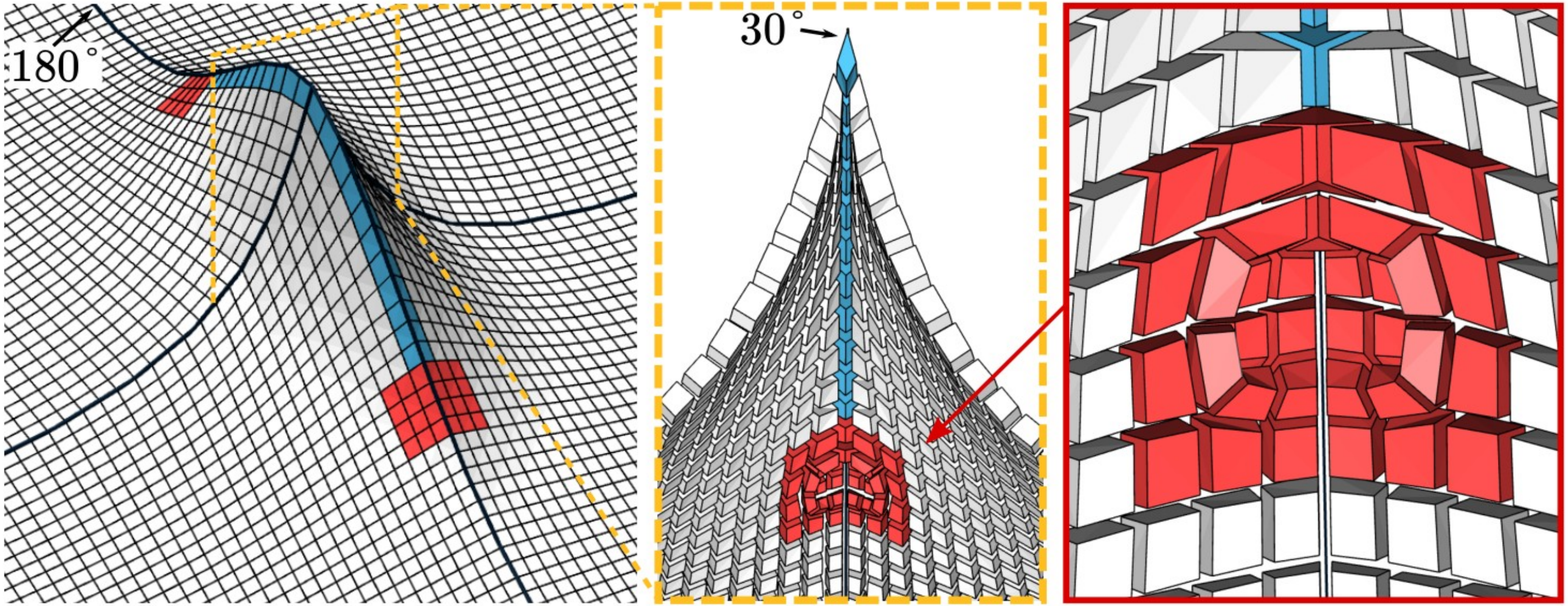
$$\underset{\mathbf{n},\ 1 \le n_i \le 4}{\text{minimize}} \quad \sum_{i=1..N_e} (n_i - x_i)^2$$

subject to **hex mesh topology**

with:

- ***i*** **edge** id in the input surface mesh
- n_i boundary edge **hex-valence**
- $x_i = \dfrac{\alpha_i}{\pi/2}$ ideal valence (α_i surface dihedral angle)

Floating-point ideal valences:
90° → x_i = 1
180° → x_i = 2
270° → x_i = 3
110° → x_i = 1.22



Example: optimal hex layer solution on pinched surface

Optimization problem reformulation

$$\underset{\mathbf{n},\ 1 \le n_i \le 4}{\text{minimize}} \sum_{i=1..N_e} (n_i - x_i)^2$$

subject to **hex mesh topology**

Floating-point ideal valences:
181° → x_i = 2.01
110° → x_i = 1.22

Integer unknowns (edge hex-valence)

How to translate "hex mesh topology" into workable constraints ?

Duality between local boundary hex config and disk triangulation

Liu et al. 2018:

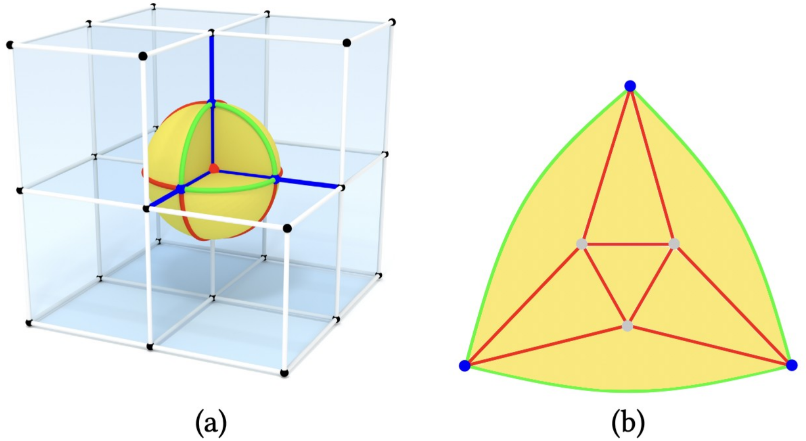
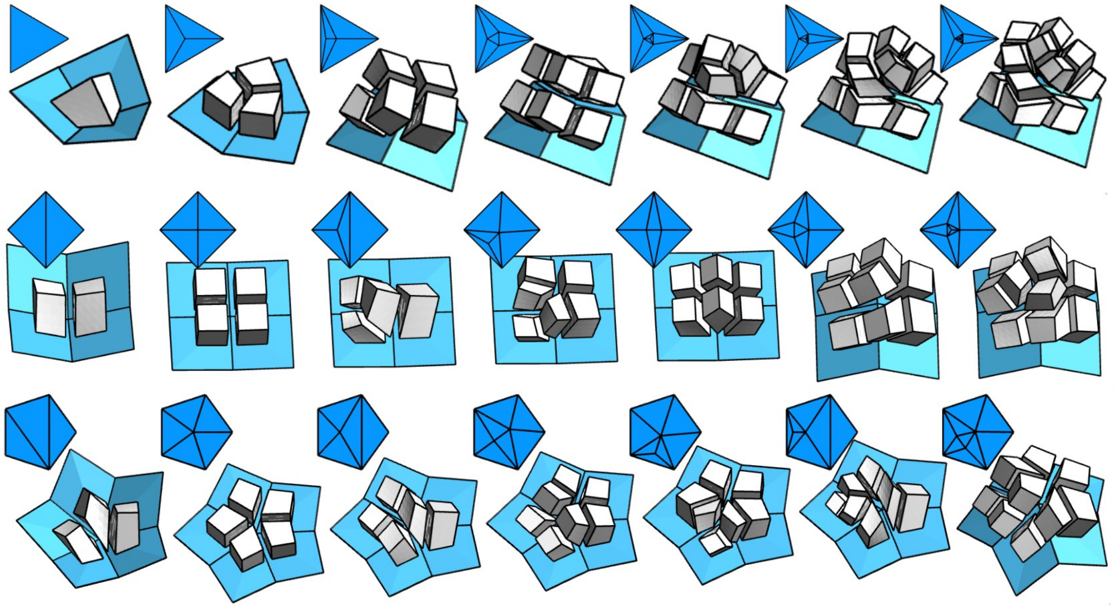


Fig. 7. (a) Boundary singular vertex intersected with a yellow hemisphere. The green boundary of the hemisphere corresponds to intersection with the hex mesh boundary. (b) Triangulation of the hemisphere, with Vertices correspond to intersections between hex mesh edges and the hemisphere.



All possible boundary hex configurations
↔ all possible disk triangulations

Problem reformulation in terms of disk triangulations

$$\underset{\mathbf{n},\ 1 \le n_i \le 4}{\text{minimize}} \quad \sum_{i=1..N_e} (n_i - x_i)^2$$

subject to **hex mesh topology**

with:

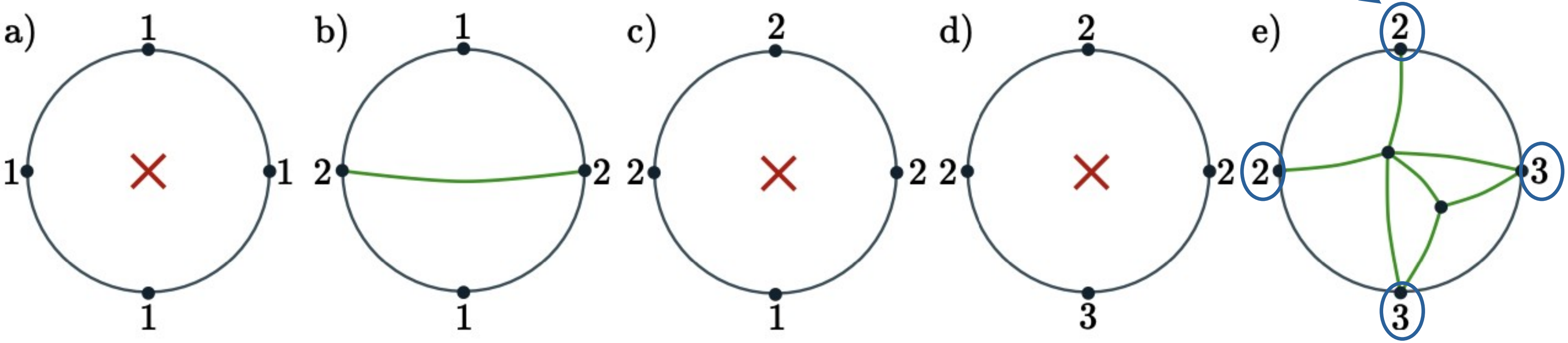
n_i boundary edge hex-valence

$x_i = \frac{\alpha_i}{\pi/2}$ ideal valence (α dihedral angle)

$$\underset{\mathbf{n},\ 1 \le n_i \le 4}{\text{minimize}} \quad \sum_{i=1..N_e} (n_i - x_i)^2$$

subject to $\forall j \in [1..N_v],\ \exists \mathcal{DT}^j(\mathbf{n}^j)$

For each boundary vertex **j**,
there **exists a disk triangulation** matching the prescribed boundary valences $\mathbf{n}^j=(n_1, \ldots, n_m)$



Integer formulation of the constrained disk triangulation problem

PROPOSITION 1. *A triangulation of the disk $\mathcal{DT}(\mathbf{n})$ whose boundary vertex valences match the set $\mathbf{n} = (n_1, ..., n_m)$ exists if and only if one has, for all reduced problem $\mathbf{n}^s = (n_1^s, ..., n_k^s)$ recursively obtained by ablation of valence one vertices*

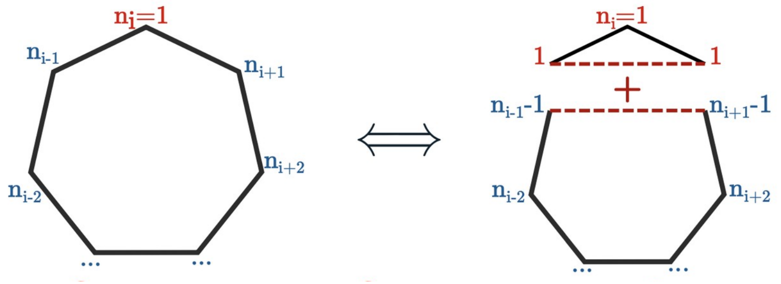
$$\#(n_i^s = 2) \neq k - 1$$

$$k > 3, n_i^s = 1 \implies n_{i-1}^s \neq 1 \text{ and } n_{i+1}^s \neq 1$$

$$k = 3 \implies \#(n_i^s = 1) = 0 \text{ or } 3$$

Conditional and **counting** integer constraints

Recursive vertex ablation:



Concrete example of integer constraints for regular quad vertex (4-valent):

$$\#(n_i = 2) \neq 3$$

$$n_1 = 1 \implies \#(n_i^{s2} = 2) \neq 2 \text{ and } \#(n_i^{s2} = 1) \neq 1 \text{ and } \#(n_i^{s2} = 1) \neq 2$$

$$n_2 = 1 \implies \#(n_i^{s3} = 2) \neq 2 \text{ and } \#(n_i^{s3} = 1) \neq 1 \text{ and } \#(n_i^{s3} = 1) \neq 2$$

$$n_3 = 1 \implies \#(n_i^{s4} = 2) \neq 2 \text{ and } \#(n_i^{s4} = 1) \neq 1 \text{ and } \#(n_i^{s4} = 1) \neq 2$$

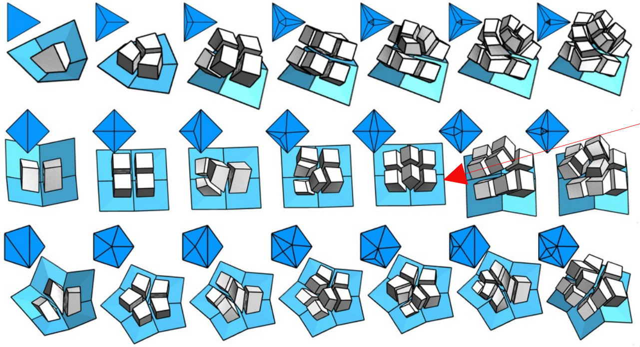
$$n_4 = 1 \implies \#(n_i^{s5} = 2) \neq 2 \text{ and } \#(n_i^{s5} = 1) \neq 1 \text{ and } \#(n_i^{s5} = 1) \neq 2$$

Global all-hex boundary layer integer problem

$$\underset{\mathbf{n},\ 1\le n_i\le 4}{\text{minimize}} \quad \sum_{i=1..N_e} (n_i - x_i)^2$$

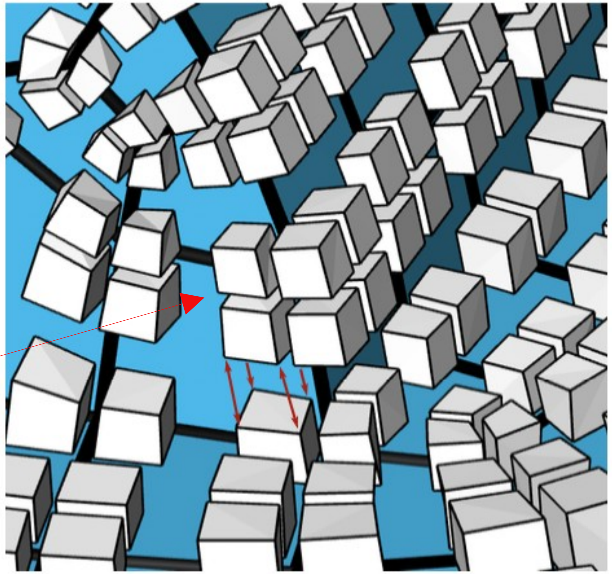
$$\text{subject to} \quad \forall j \in [1..N_v],\ \exists \mathcal{DT}^j(\mathbf{n}^j)$$

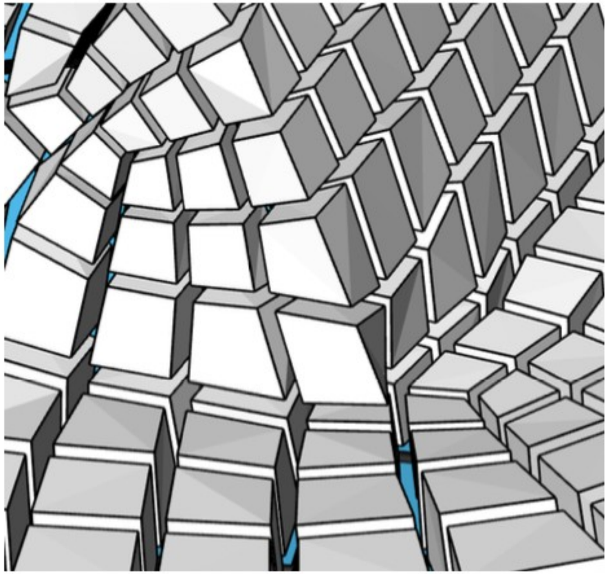
Local configs



Compatibility constraints:

- two edge-adjacent local configurations have the same edge hex-valence (n_i)
- respected by construction (one unknown per edge)



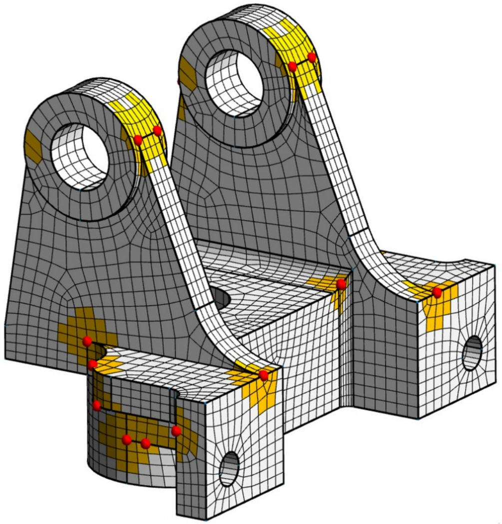


The result all-hex layer matches the **midpoint subdivision** of the input surface mesh

How to solve efficiently the global integer problem ?

- Branch and bound solver with constraint propagation
  - Highly non-linear constraints (counting and conditional)
  - We use the **Gecode** library *(Schulte et al. 2006), MIT license*
  - The problem has a guaranteed solution: **n**=2 (i.e. surface extrusion)

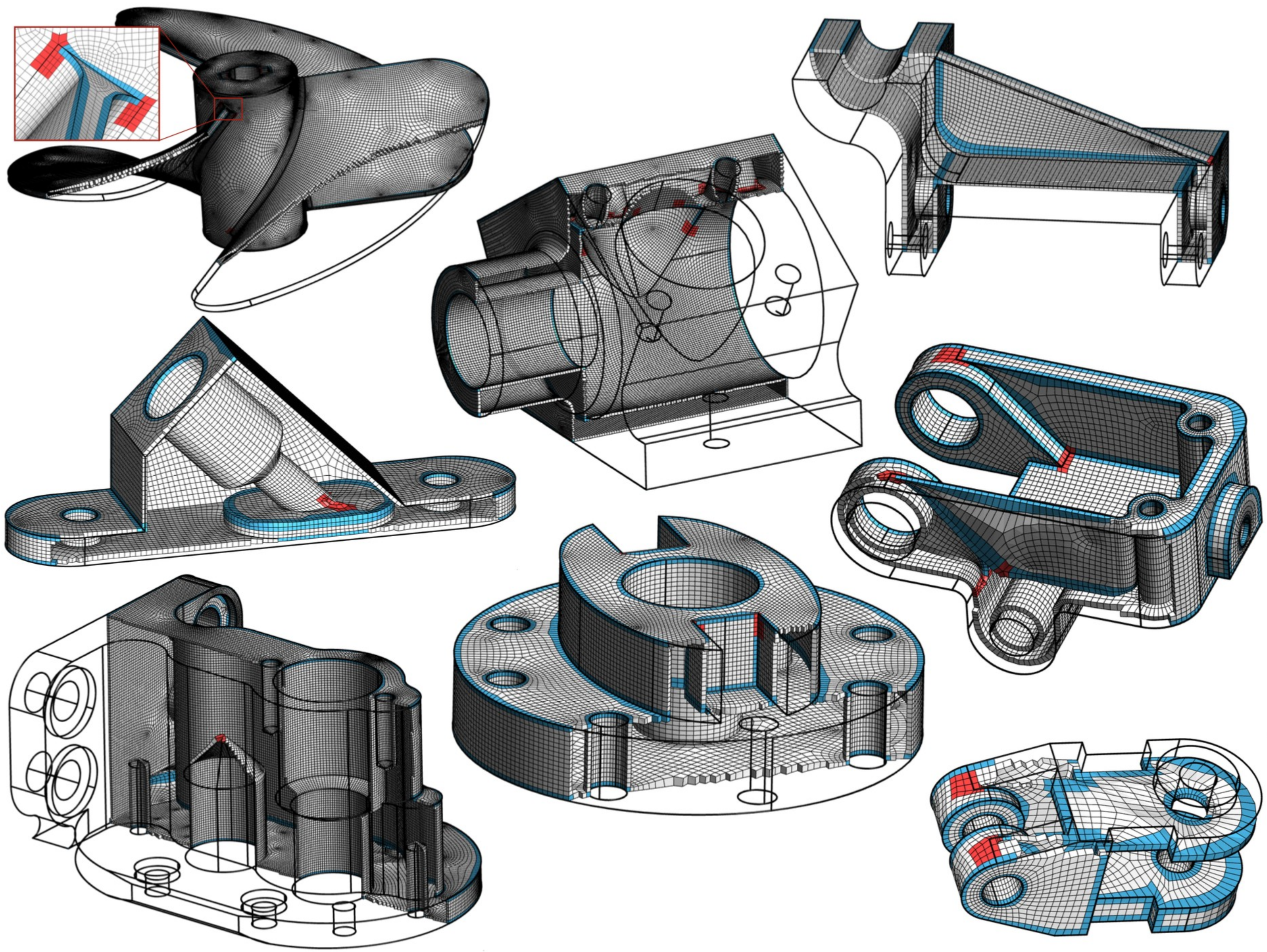
- Problem decomposition to make the problem solvable in practice



Global: 106k integer unknowns
→ 18 sub-problems with between 35 and 105 integer unknowns

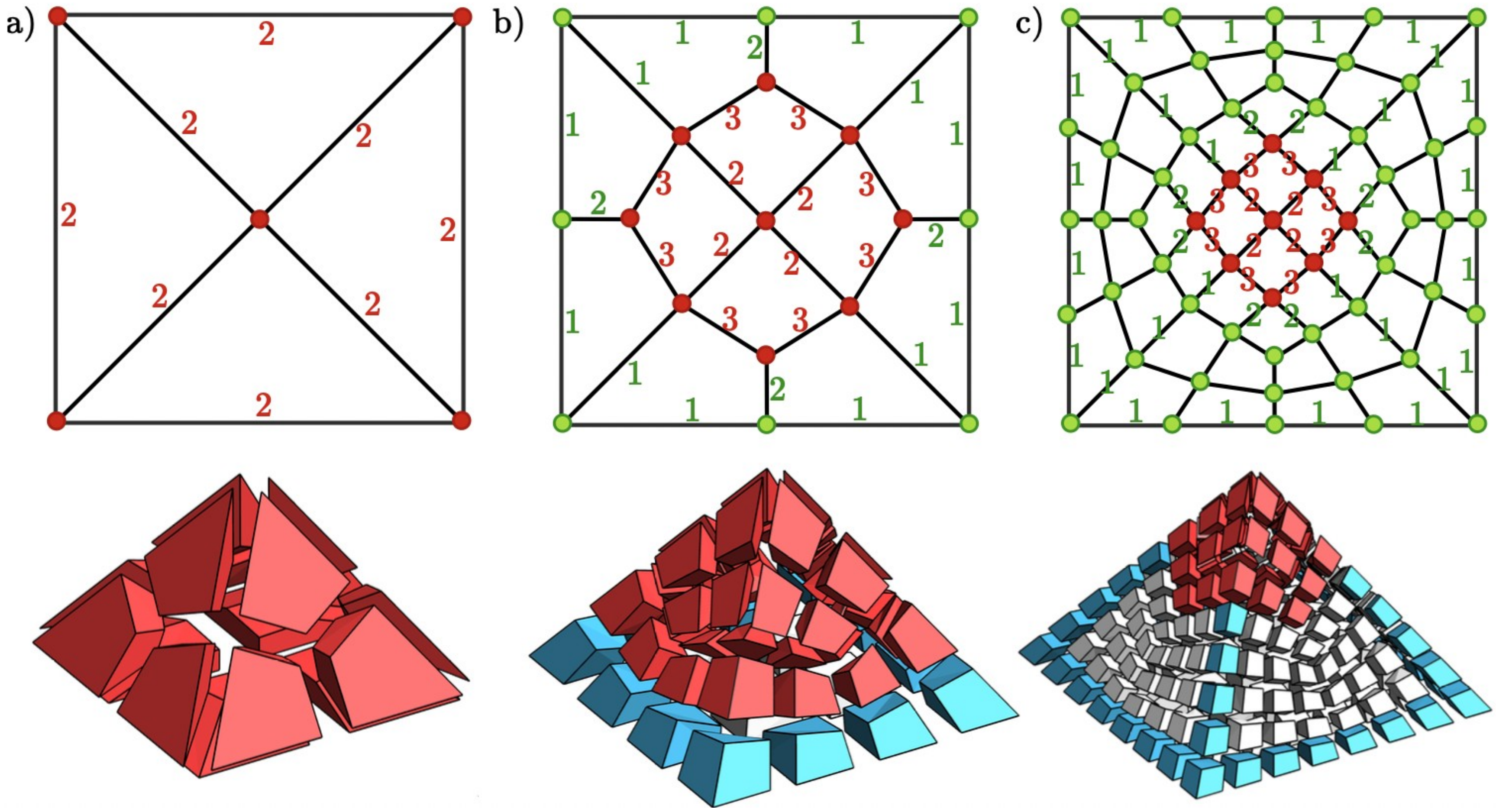
Fast: - <1 sec per sub-problem in practice
- solved in parallel

Results

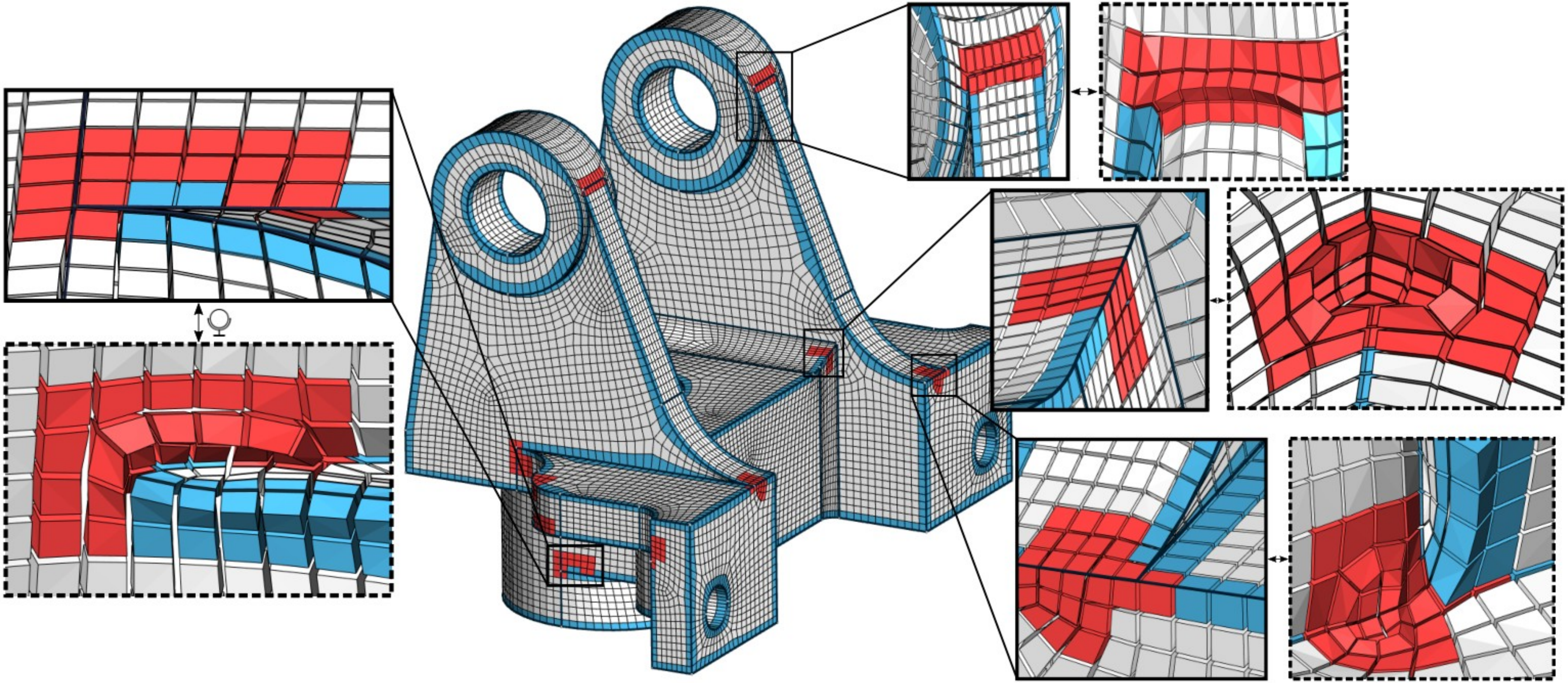


- Topological problem is 100% robust
- Generic, no hardcoded case
- ABC dataset → always better than extrusion
- Find non-intuitive configs
- Difficult configs are solved in local neighbourhoods
  → No global pollution

Example of localized solving for pyramid apex



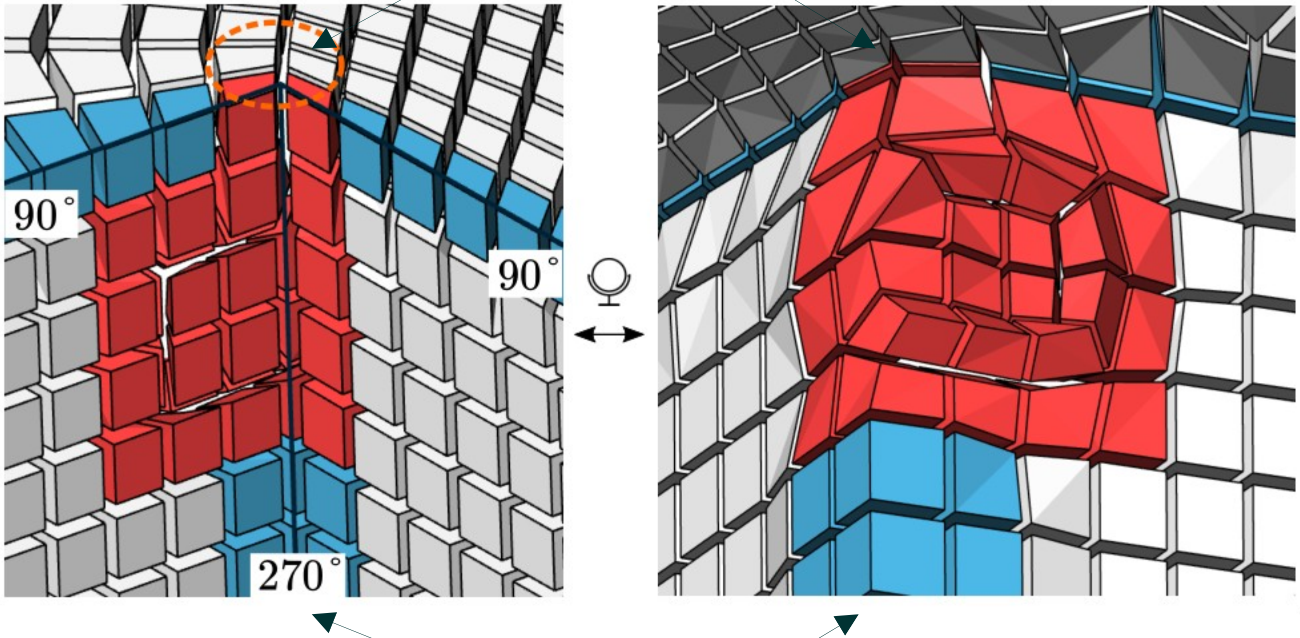
Example of local solutions found by the optimization solver



Quality will be impacted by initial surface quad mesh

Ridge/corner vertex topology

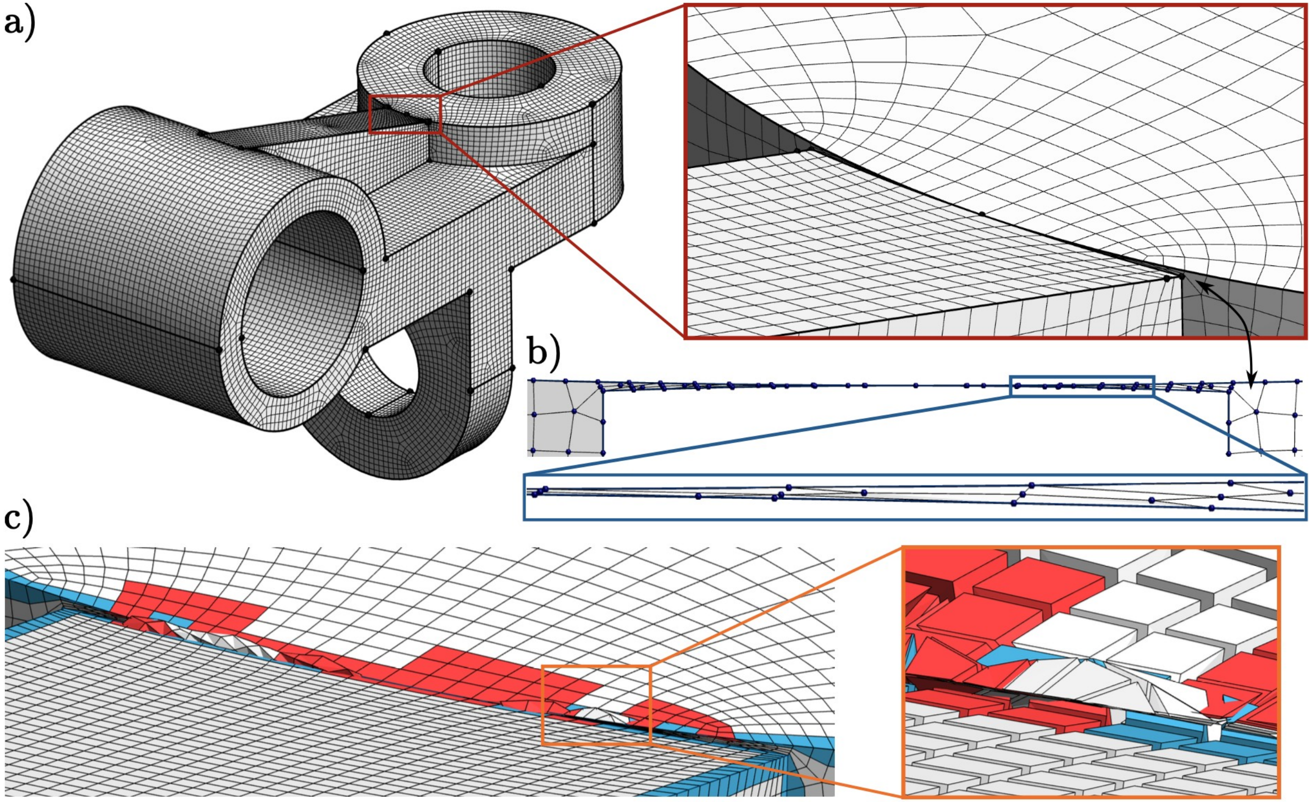
valence 2



valence 3

Geometry of the all-hex layer

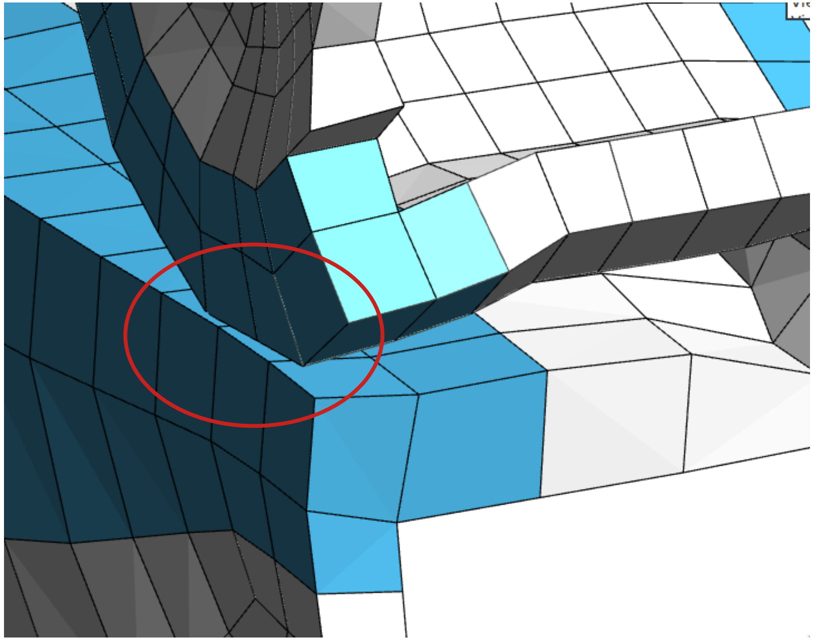
- Geometry: untangling/smoothing based on *(Garanzha et al. 2021)*
- Sometimes there is no good solution (same for surface extrusion)



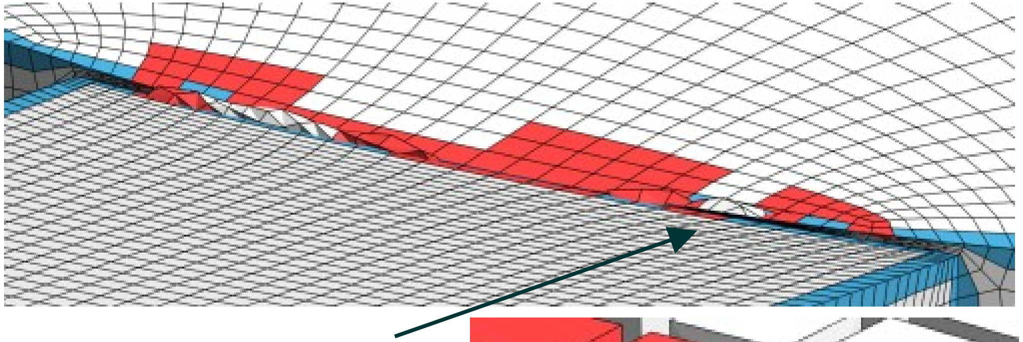
CAD acute (<1deg)
+ bad quad mesh
→ tangled hexa

Geometrical issues

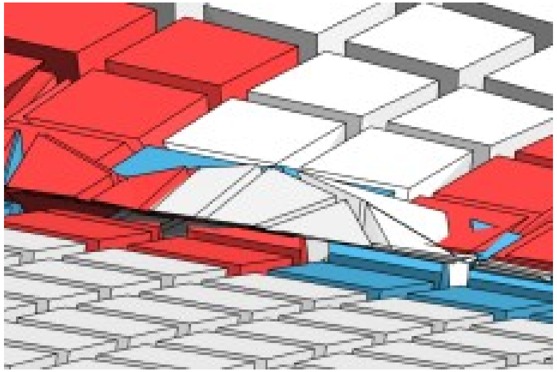
- Common issue: self-intersections of the interior quad mesh



thin region + layer too thick → overlap

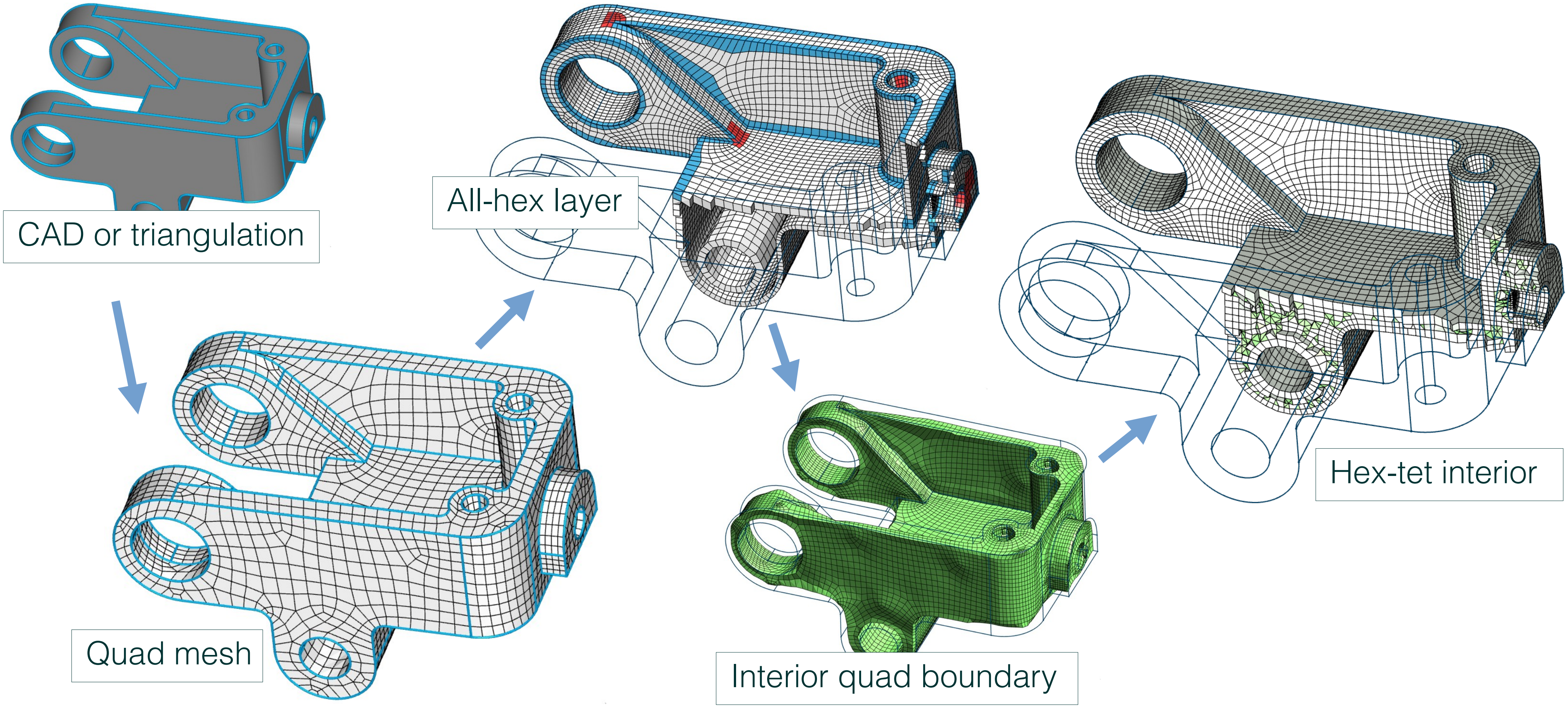


CAD acute (<1deg) + bad quad mesh → tangled hexa

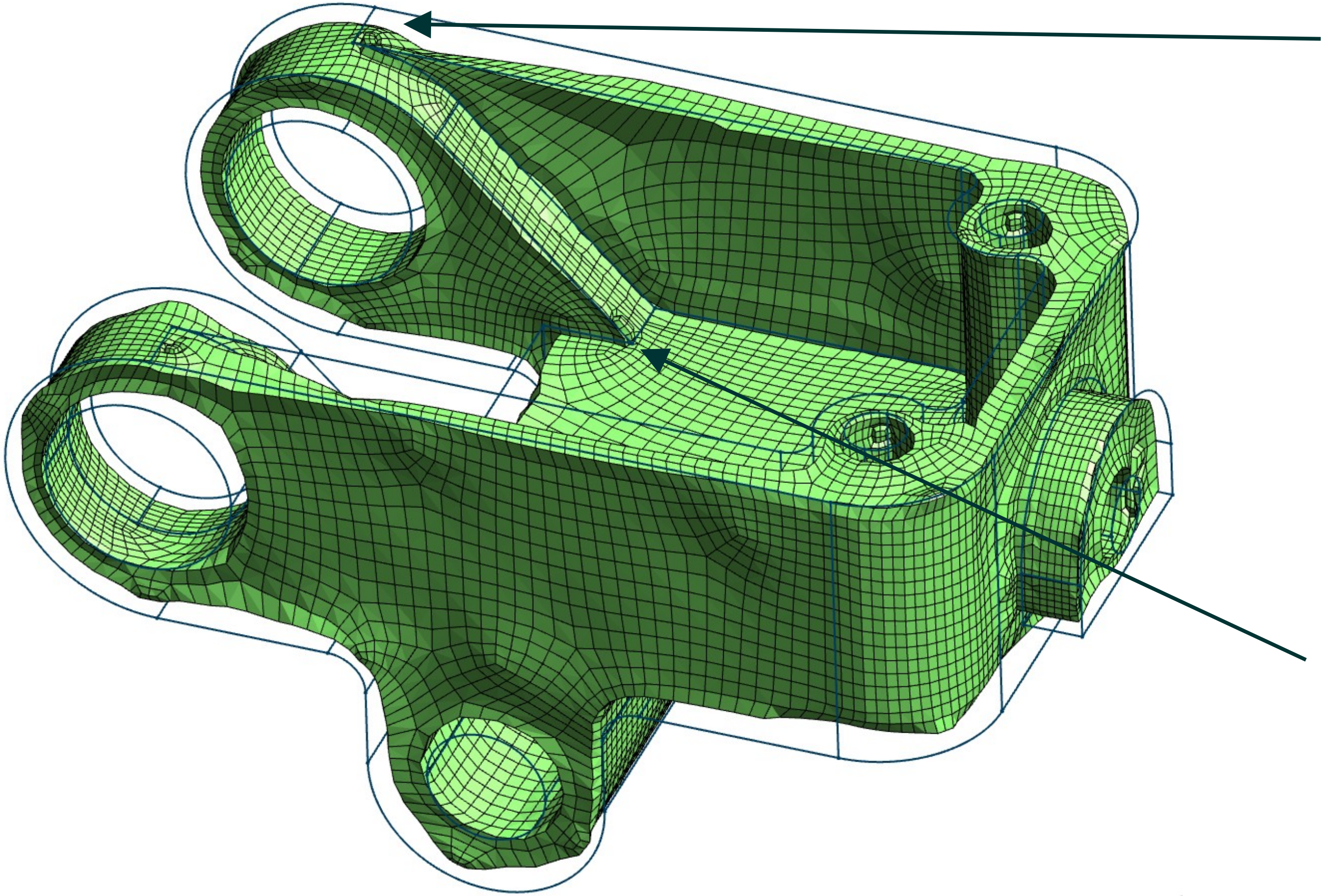


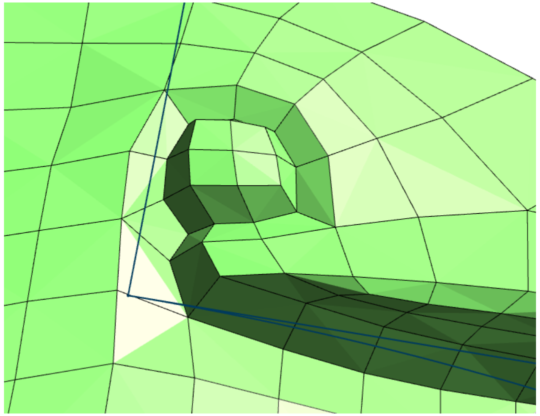
- **On dataset: 83/419 models with self-intersections** (using default parameters)

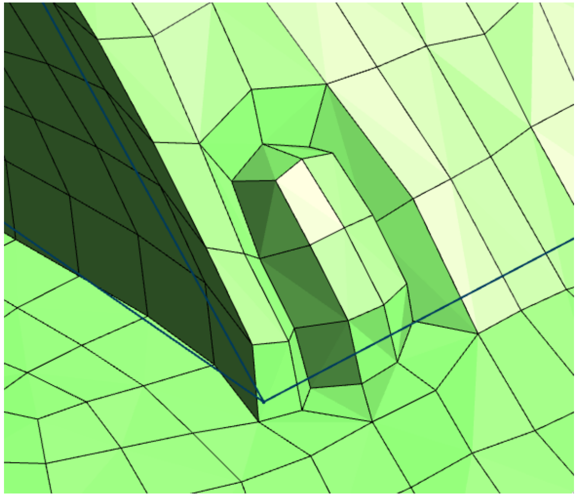
Going back to hex-dominant meshing pipeline



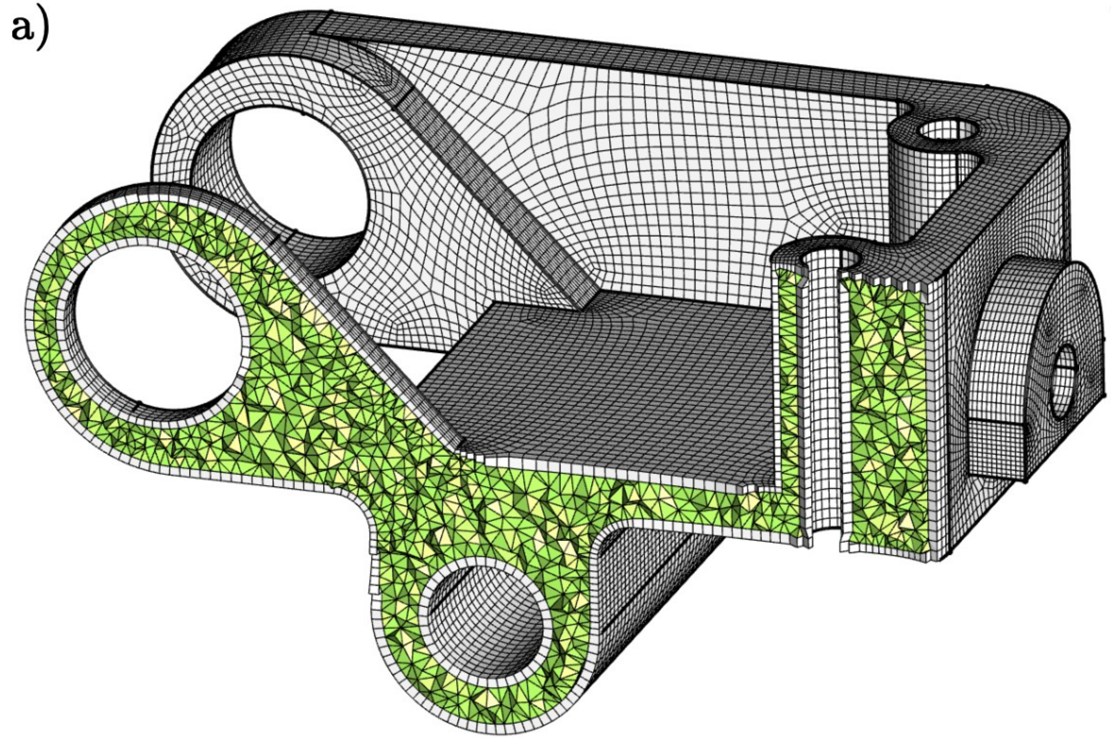
Remaining volume: interior quad boundary

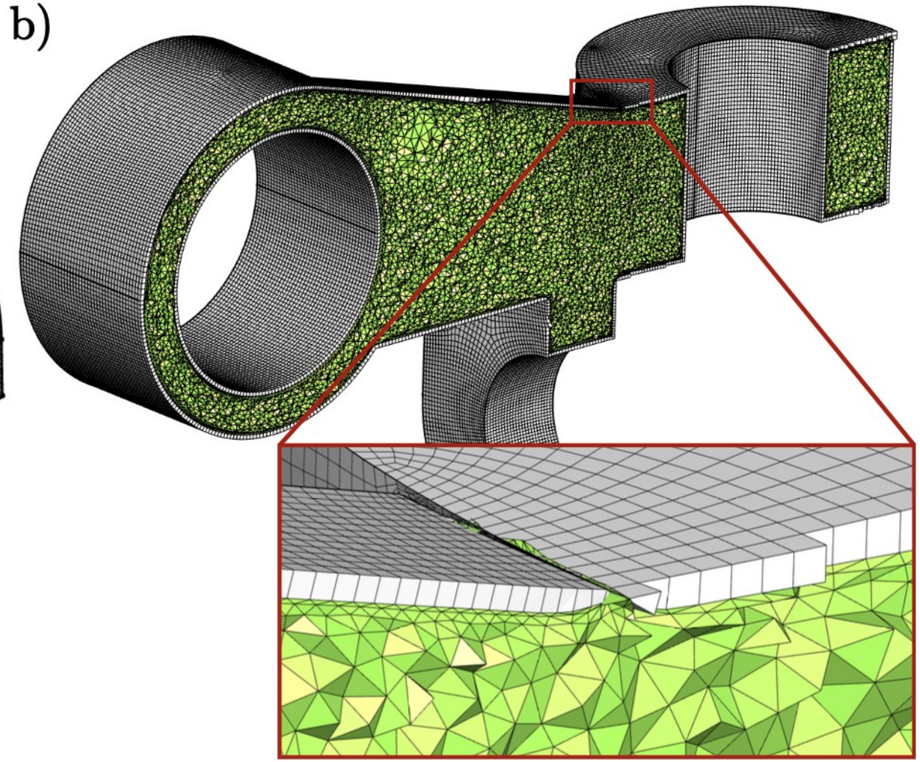






Robust topological interior tet meshing





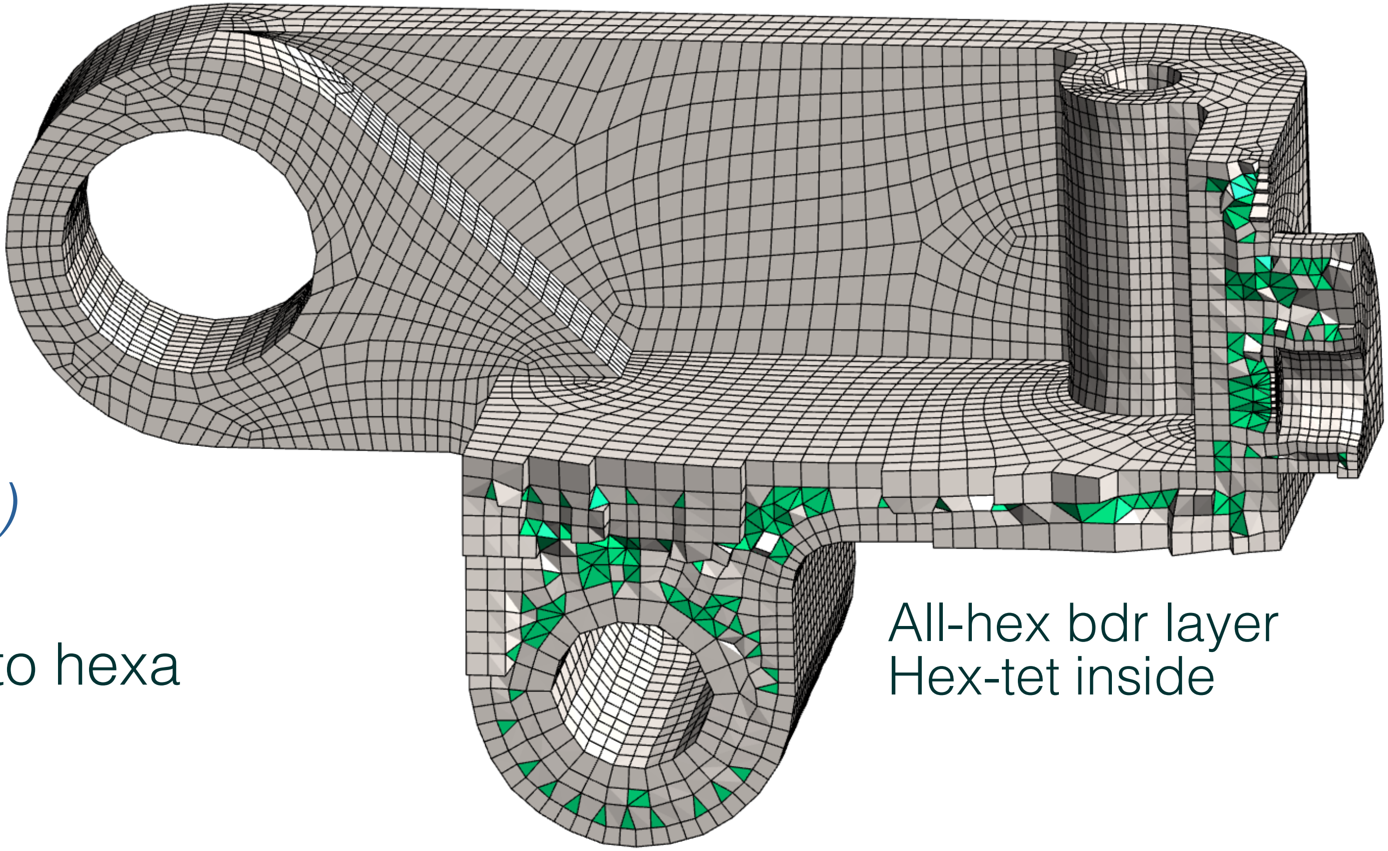
Interior quad mesh
without self-intersections:
→ constrained tet mesher

With self-intersections:
→ topological hex-tet transitions to
go back to the initial surface geometry

Hex-dominant interior mesh

- Frame-field guided frontal point insertion *(Georgiadis et al. 2021)*

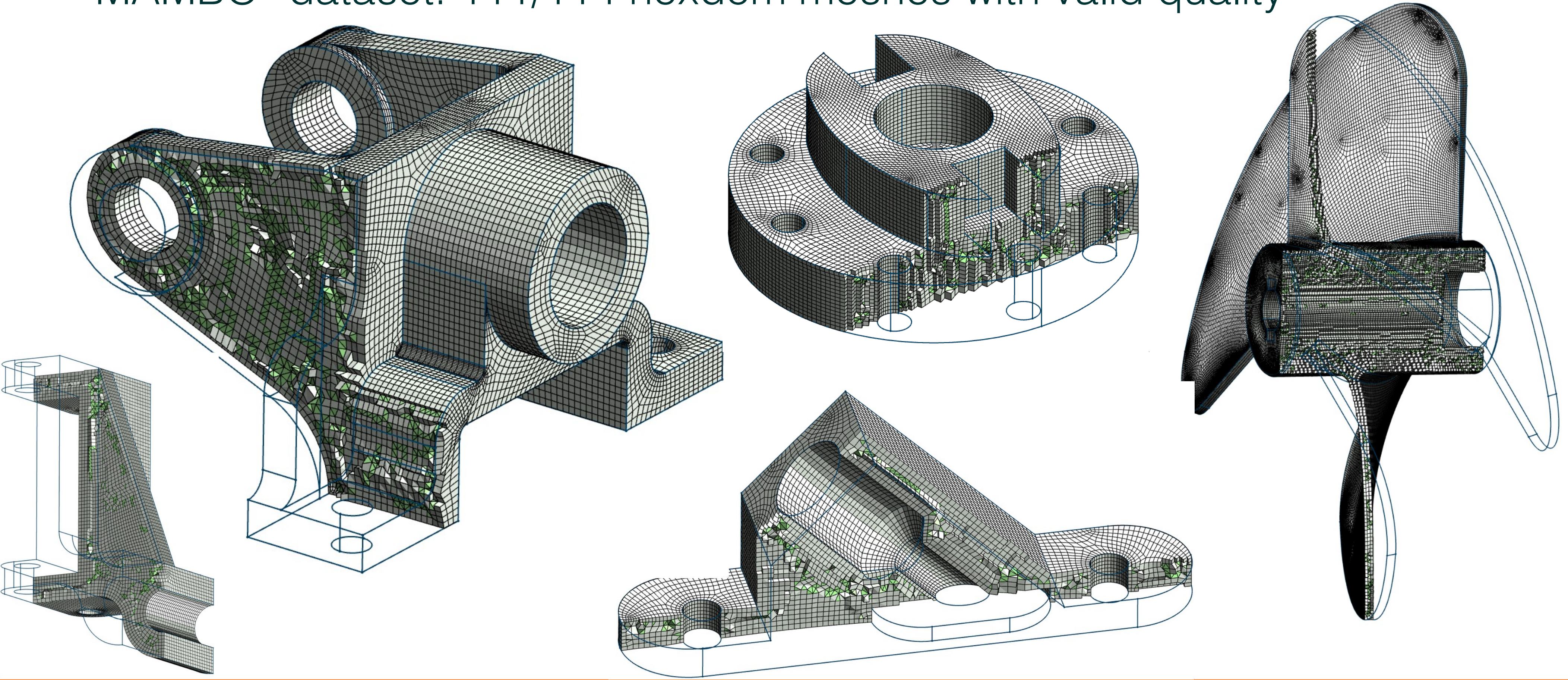
- Combination of tetra into hexa *(Pellerin et al. 2018)*



All-hex bdr layer
Hex-tet inside

Hex-dominant meshing results on Mambo

- MAMBO* dataset: 111/114 hexdom meshes with valid quality



* https://gitlab.com/franck.ledoux/mambo

Conclusions and future work

More information

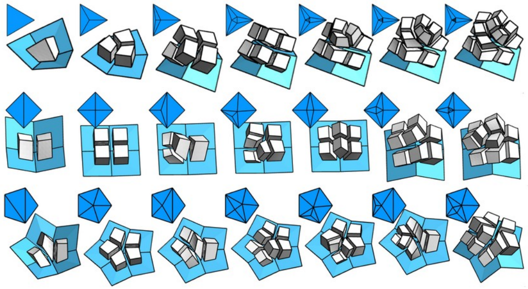
- Preprint at https://mxncr.github.io/
- Open-source code at https://gitlab.onelab.info/gmsh/gmsh/-/tree/hexbl

Key idea

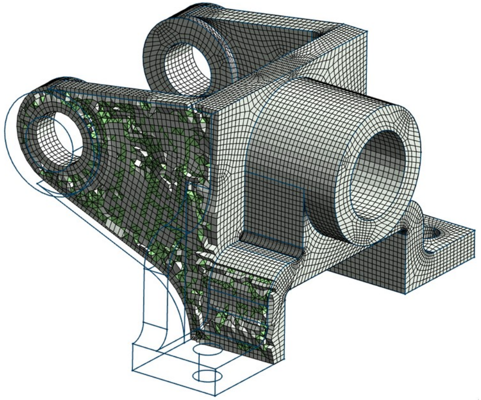
- **Exhaustive local hex configurations** can be explored thanks to duality with disk/sphere triangulations
- Disk triangulation problems (existence here) can be **reformulated into integer problems** !

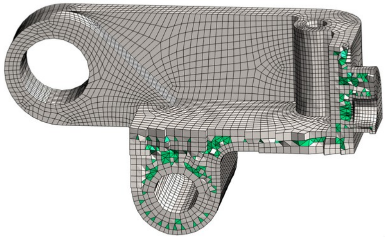
Future work

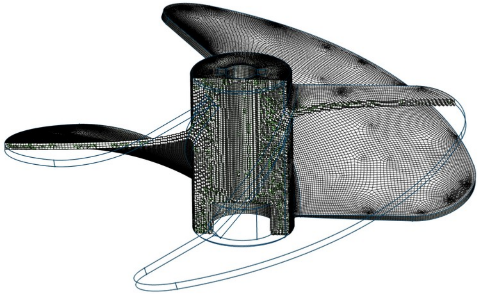
- Multiple hex layers
  - Isotropric subdivision or iterative all-hex layer
  - Anisotropric subdivision with non-hex at irregular configs
- Control over layer thickness
- For hex-dominant: fix the tet mesh quality (if invalid due to topological transition)



Thank you for you attention
Questions ?





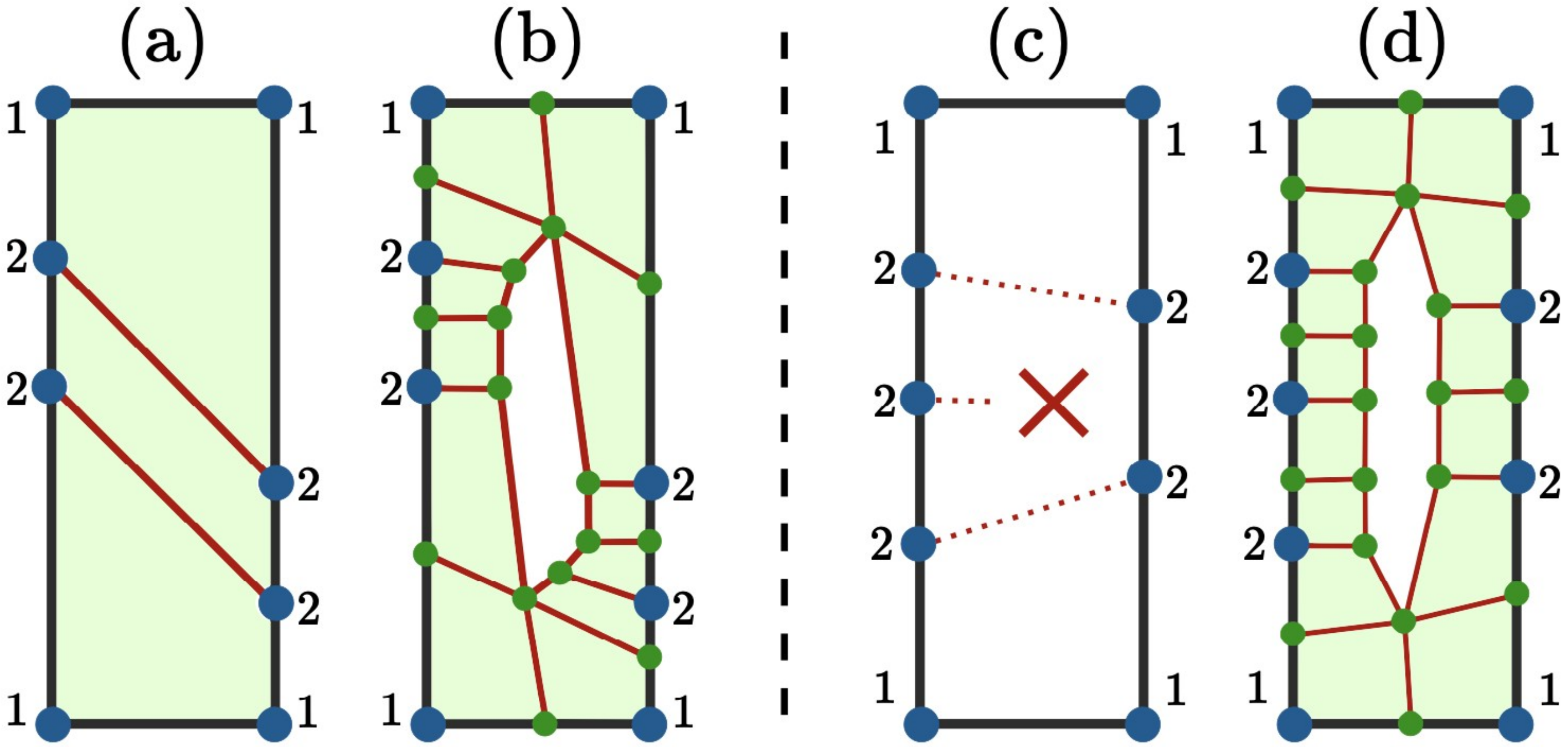


Job ads:

- Siemens Star-CCM+ meshing team have two open positions
- More information on Discord, don’t hesitate to contact me

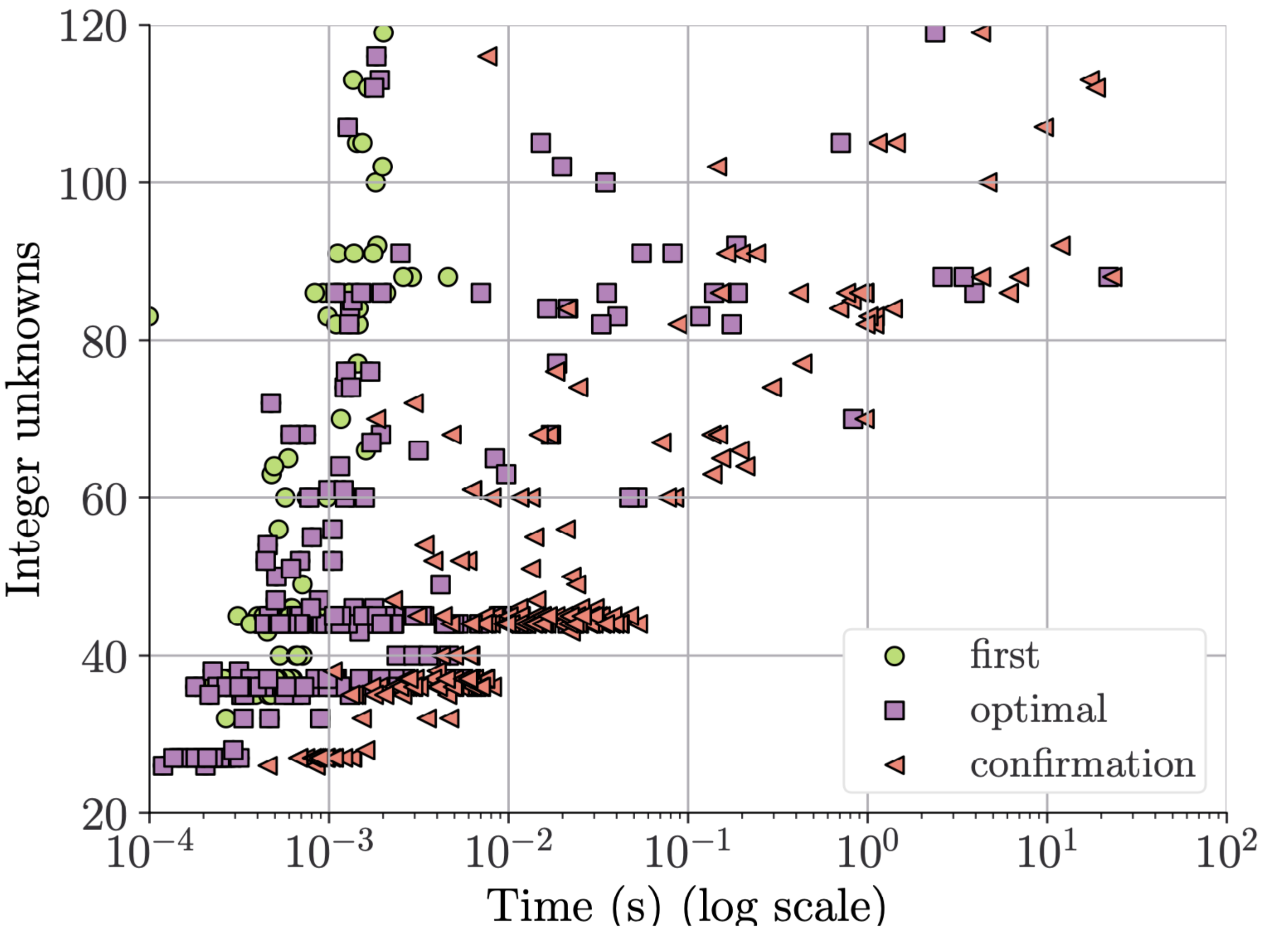
Appendix

Can we avoid midpoint subdivision ? Short answer: no



(a) and (c): without midpoint subdivision
(b) and (d): with midpoint subdivision

Performance on sub-problems



~ 0.7 sec / sub-problem

can be solved in parallel

→ solving the topological problem is fast enough