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Multiple approaches to frame field correction for CAD models

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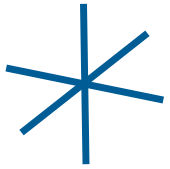
28th International Meshing Roundtable

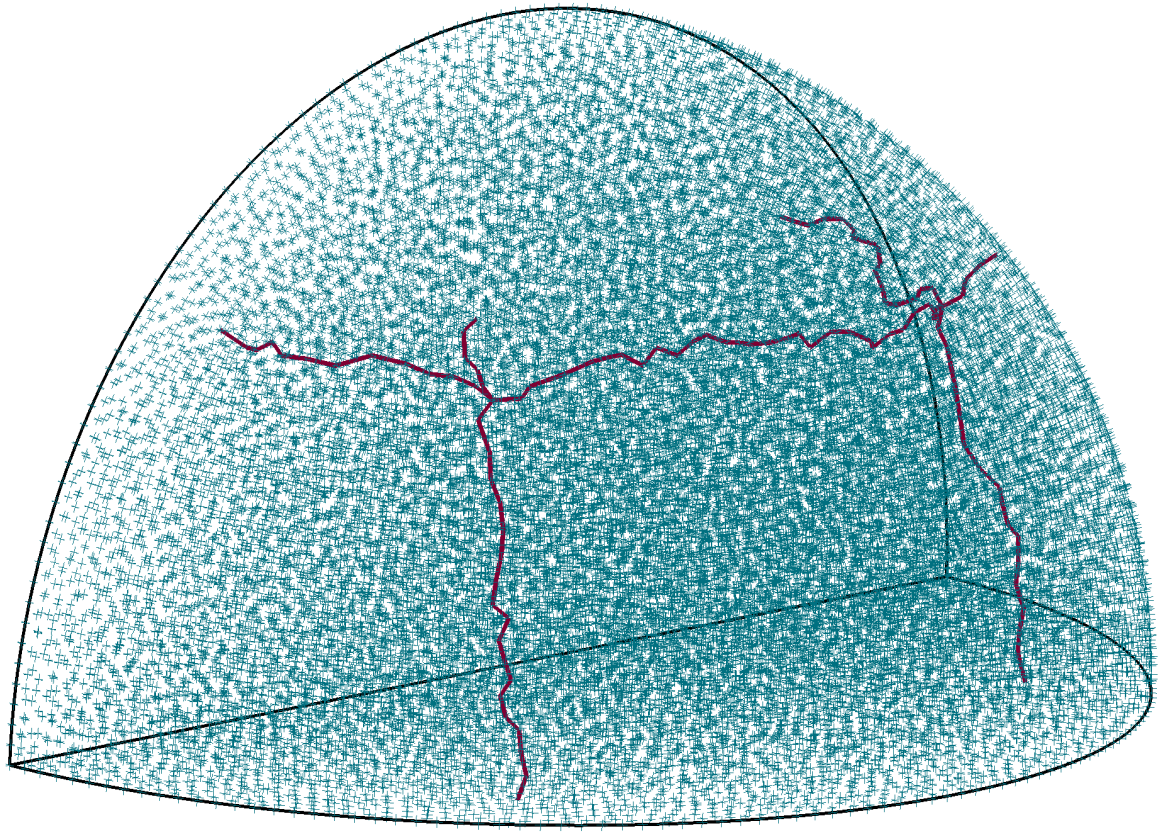
14-17 October 2019, Buffalo, USA

Boundary-aligned frame fields for full hex meshing of CAD models

2D cross:

3D cross or frame:





$$\min \int_\Omega ||\nabla \mathbf{f}||^2$$
$$\mathbf{f} \;//\; \mathbf{n} \text{ on } \partial\Omega$$

Main ideas:

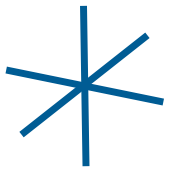
frame field $\Leftrightarrow$ field of infinitesimal cubes

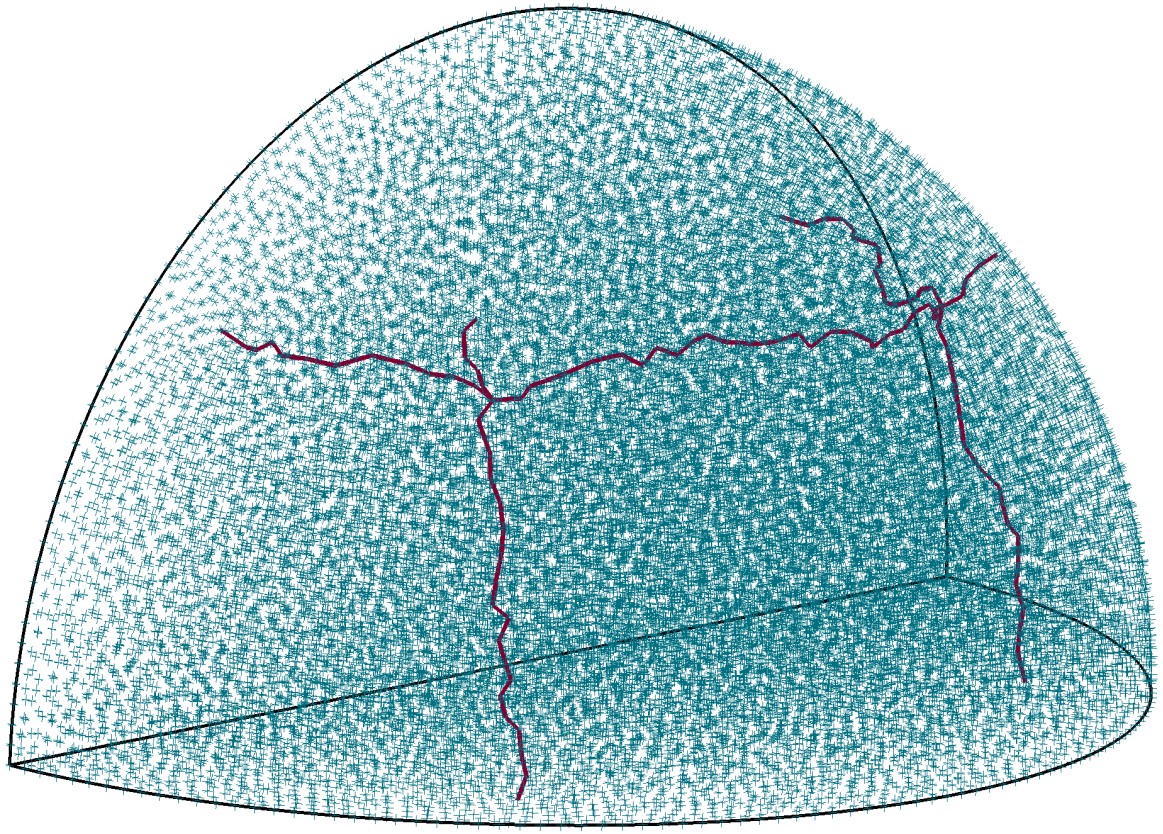
frame field singular curves $\Leftrightarrow$ irregular edges of block decomposition

Boundary-aligned frame fields for full hex meshing of CAD models

2D cross:

3D cross or frame:





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Main ideas:

frame field $\Leftrightarrow$ field of infinitesimal cubes

frame field singular curves $\Leftrightarrow$ irregular edges of block decomposition

Local geometry

Global topology

How to compute a 3D frame field from scratch ?

Minimize the Dirichlet energy

$$\min \int_\Omega ||\nabla \mathbf{f}||^2$$

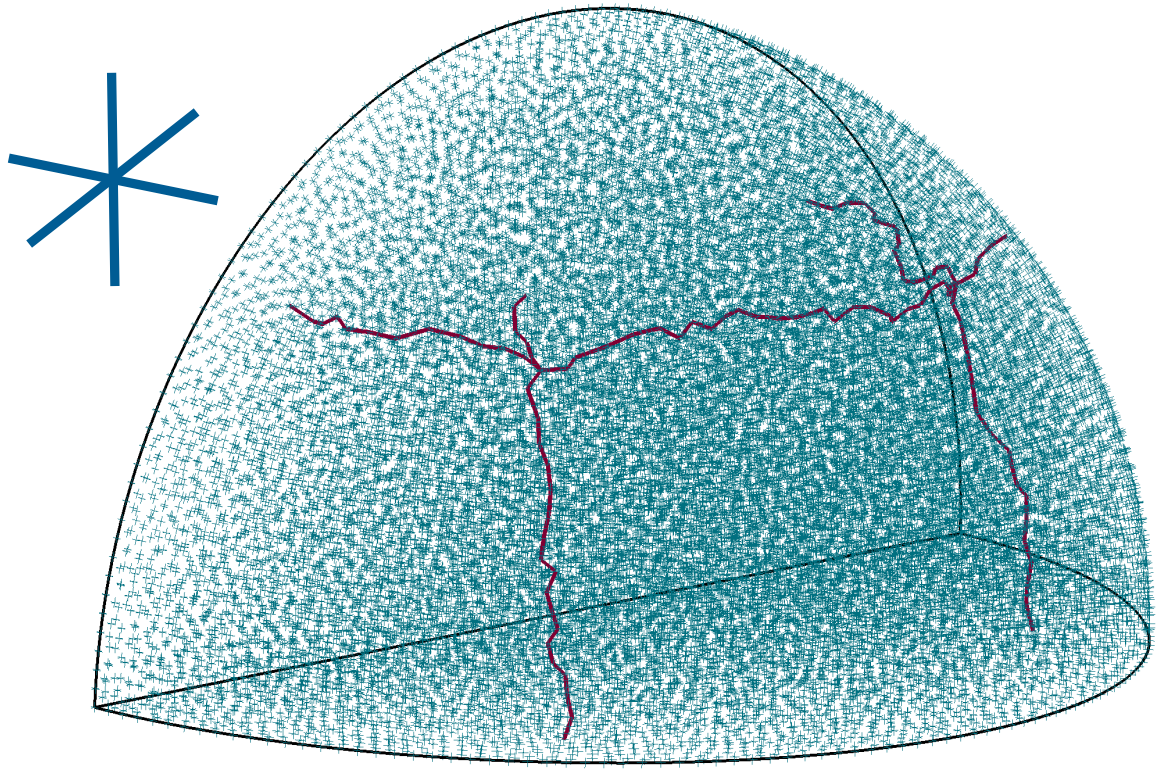
Tangency boundary conditions

$$\mathbf{f} \ // \ \mathbf{n} \text{ on } \partial\Omega$$

Space of frames is not simple :

$$\mathcal{F} = SO(3)/O$$

(with O the octahedral group)



Still an active research topic:

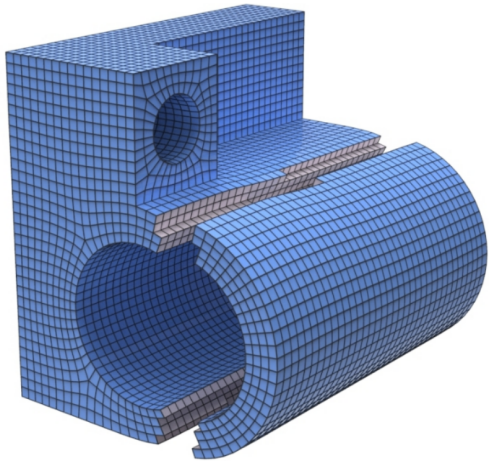
Huang et al. 2011, Li et al. 2012, Ray et al. 2016, Solomon et al. 2017.,
Chemin et al. 2018, Palmer et al. 2019, Golovaty et al. 2019, etc

When the singularity graph is known: *Liu et al. 2018, Corman et al. 2019*

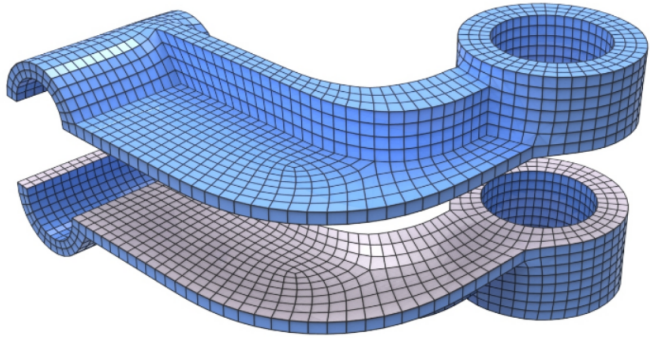
Given a frame field, how to build a block-structured hexahedral mesh ?

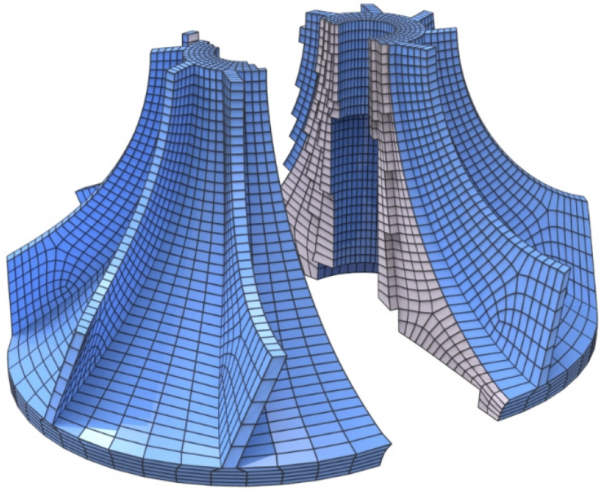
CubeCover parametrization (mixed-integer problem) + hex extraction:

Nieser et al. 2011
Li et al. 2012
Lyon et al. 2016
and others



Li et al. 2012

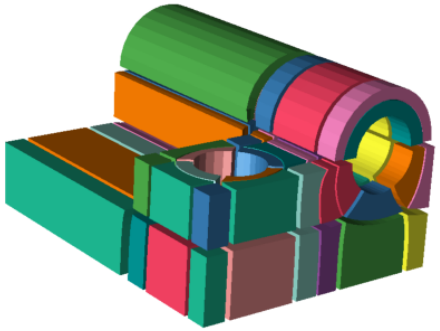




Dual surface construction + primalization:

Zheng et al. 2018
Livesu et al. 2019

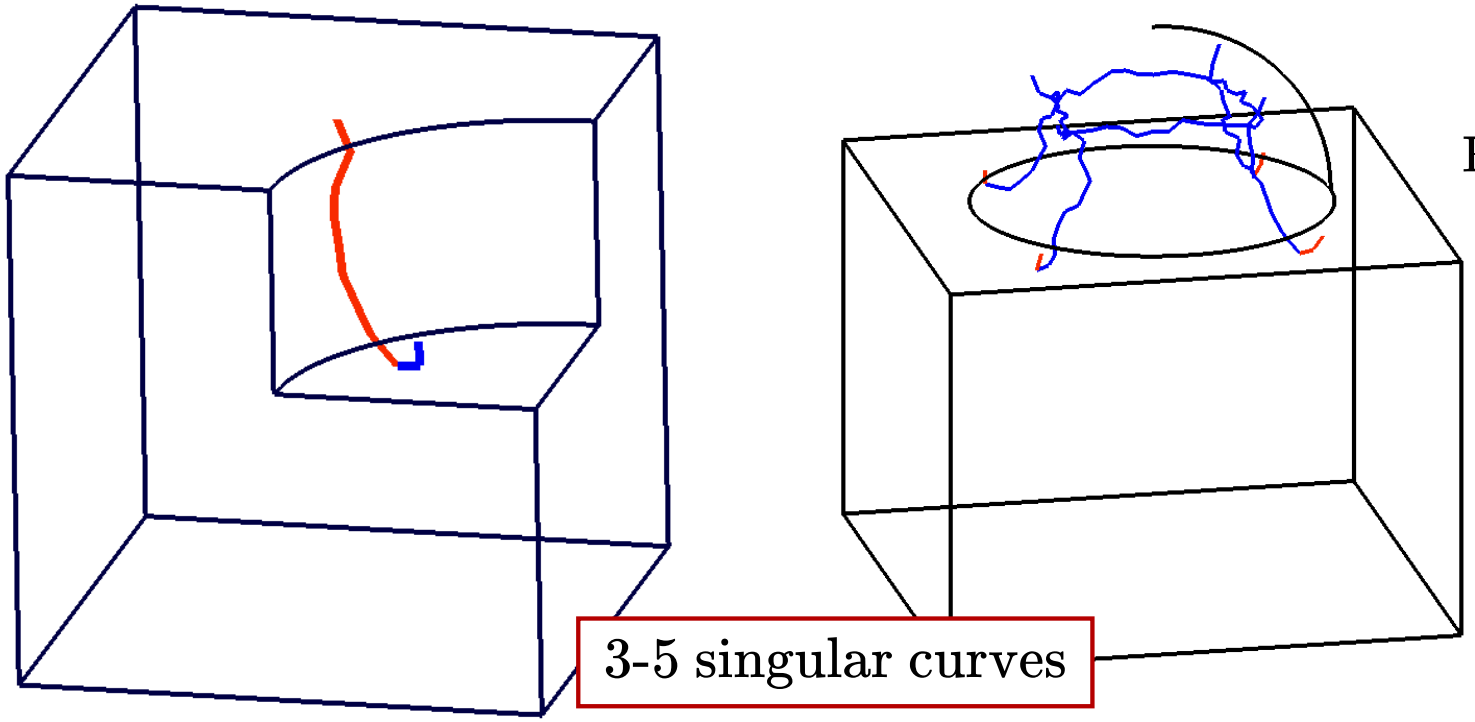
Zheng et al. 2018



... works for some models, but not robust

But some frame field singularities are not compatible with hex meshing !

State-of-the-art frame fields contain non "hex-meshable" singularities, e.g.:



3-5 singular curves

References:

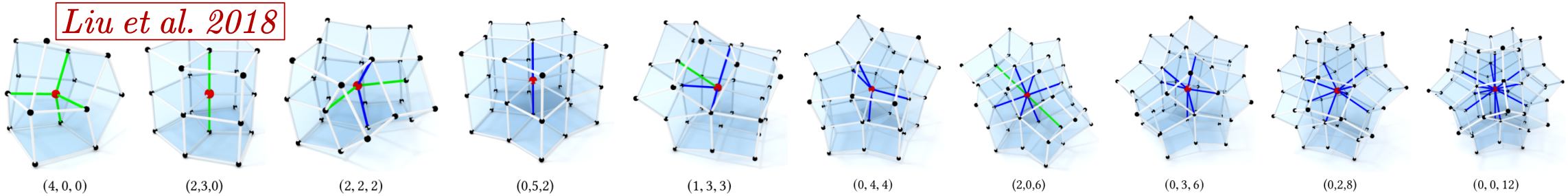
Ray and Sokolov 2015,

Viertel et al. 2016,

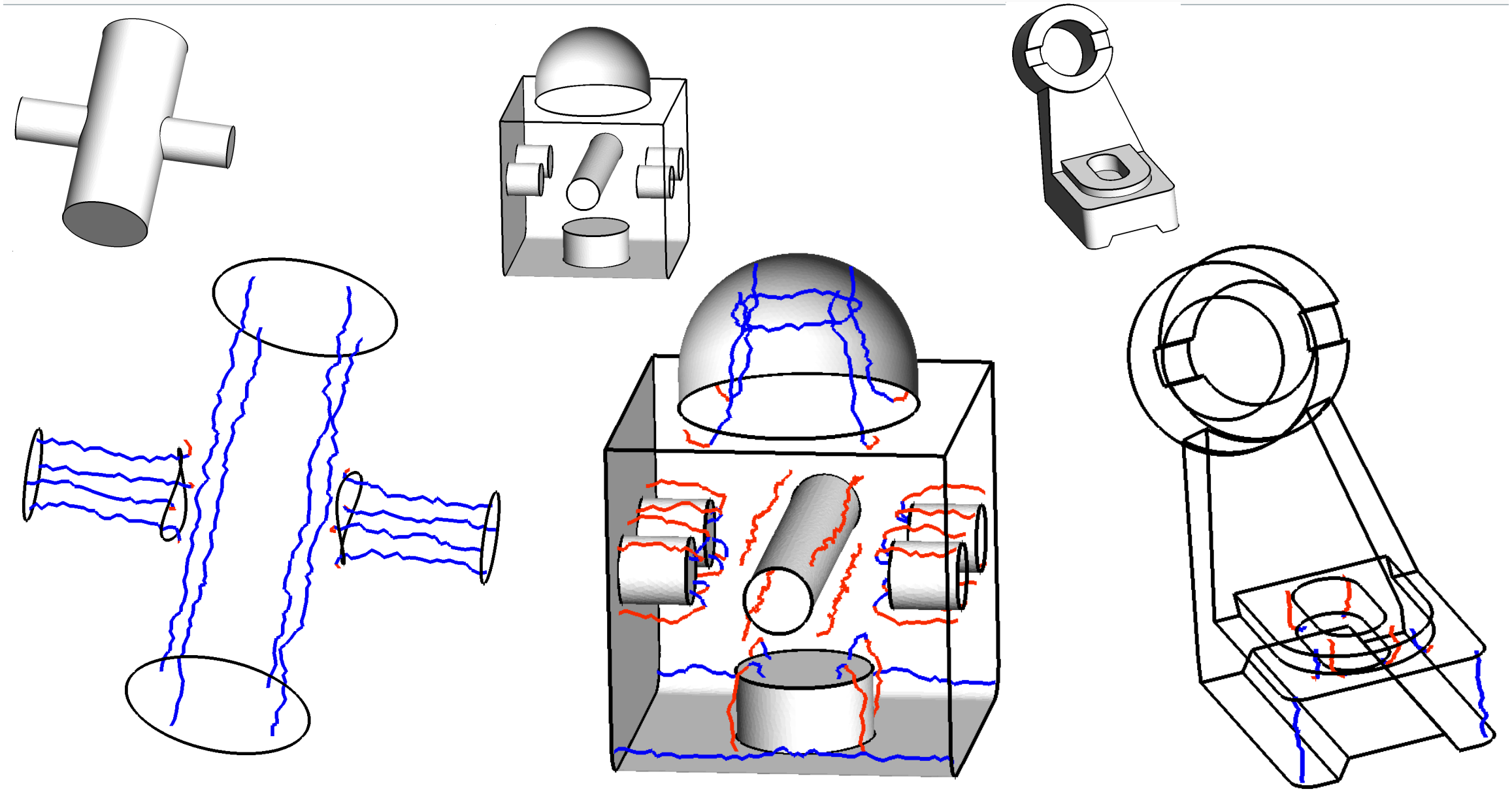
Liu et al. 2018

List of valid vertex configurations (restricted to edge valence 3, 4, 5):

Liu et al. 2018

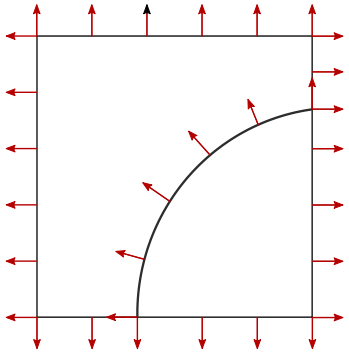


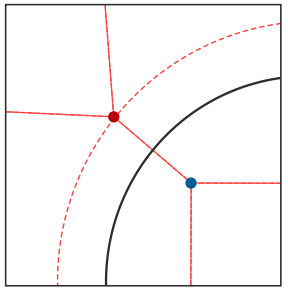
3-5 singular curves : a common issue for CAD models



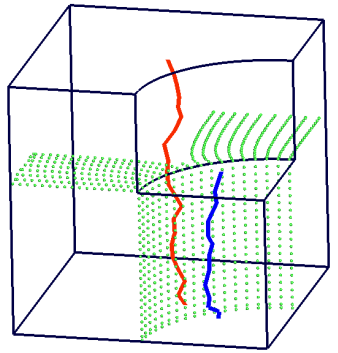
Overview - Multiple approaches to frame field correction

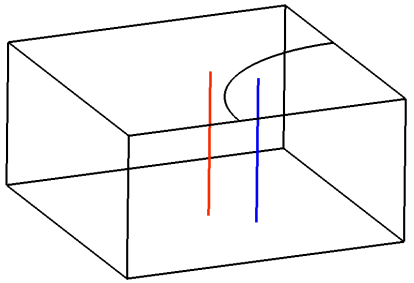
1. Issue analysis



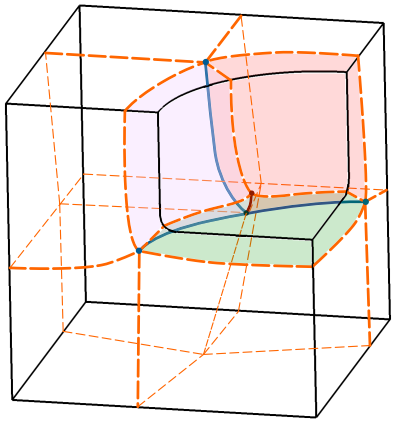


2. Extrusion of feature curves / bdr. singularilities

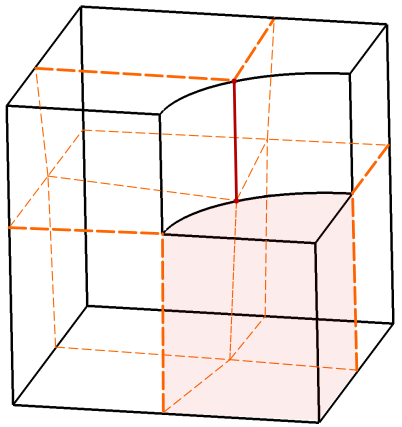




3. Smoothing of feature curves



4. Invalid singular curve boundary snapping



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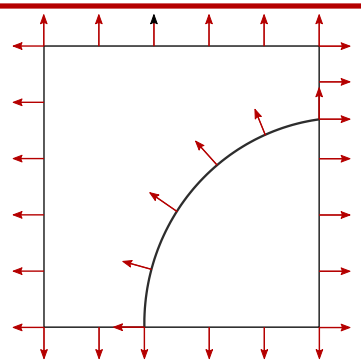

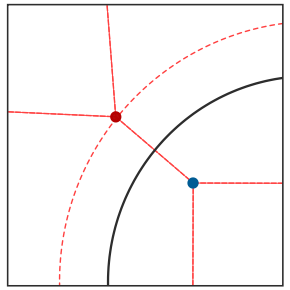

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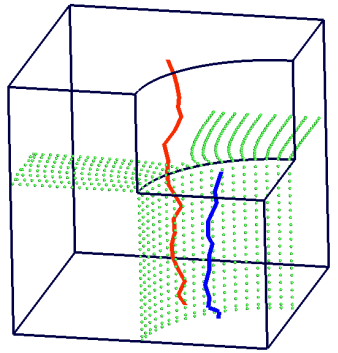

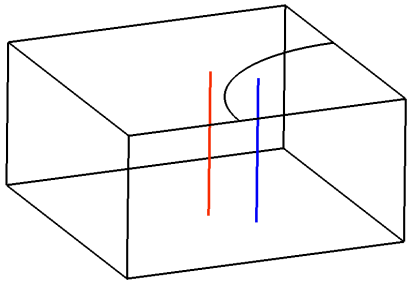

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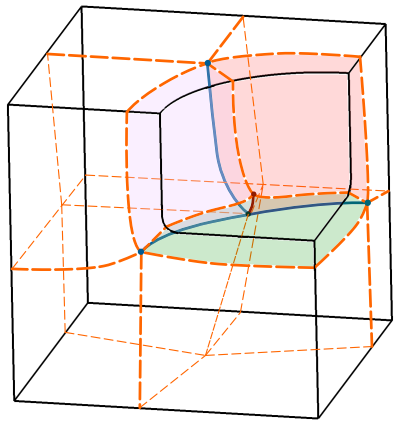

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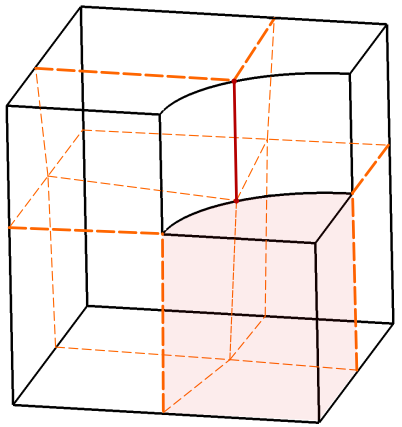
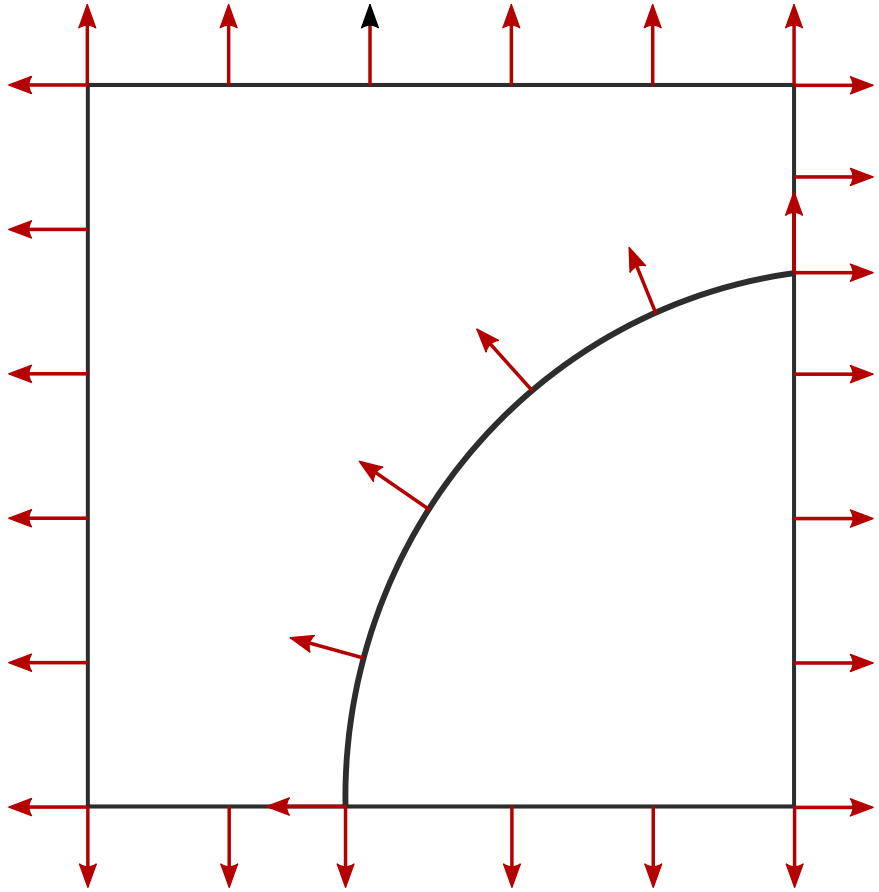


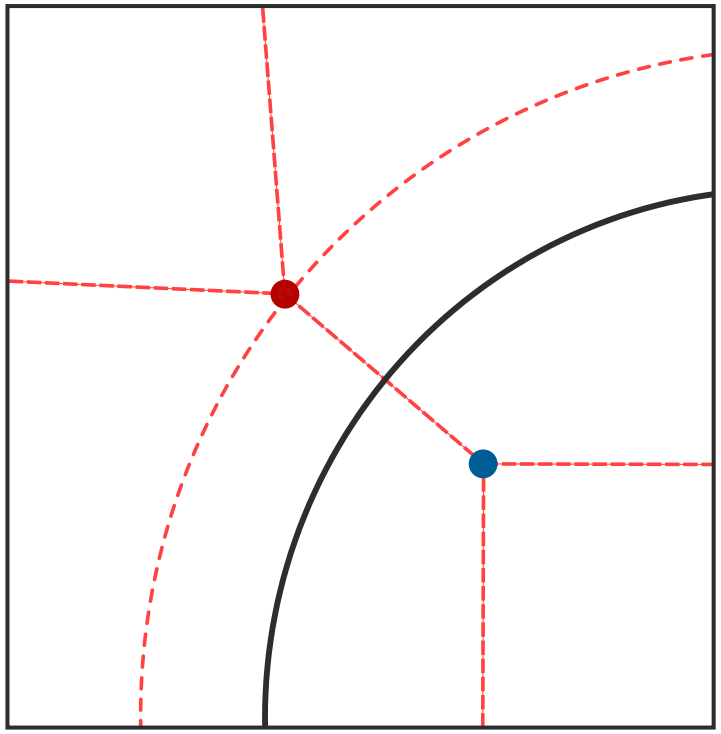
Feature curves in 2D: domain splitting

Square with imprinted arc



Boundary conditions (red arrows)

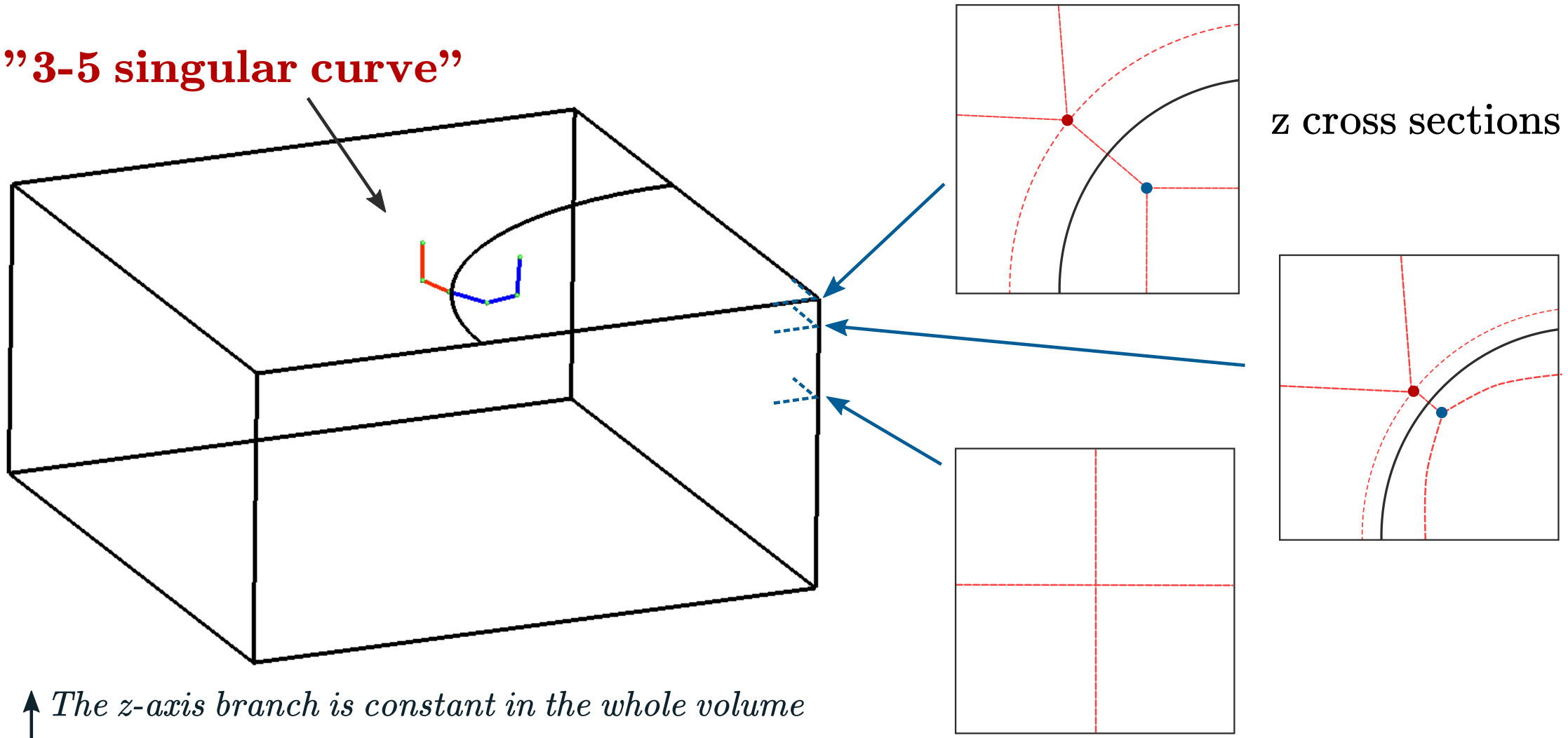
Quad decomposition (dashed lines)



Cross field singularities :

- +1/4 (valence three)
- -1/4 (valence five)

Feature curves in 3D: only the boundary is split



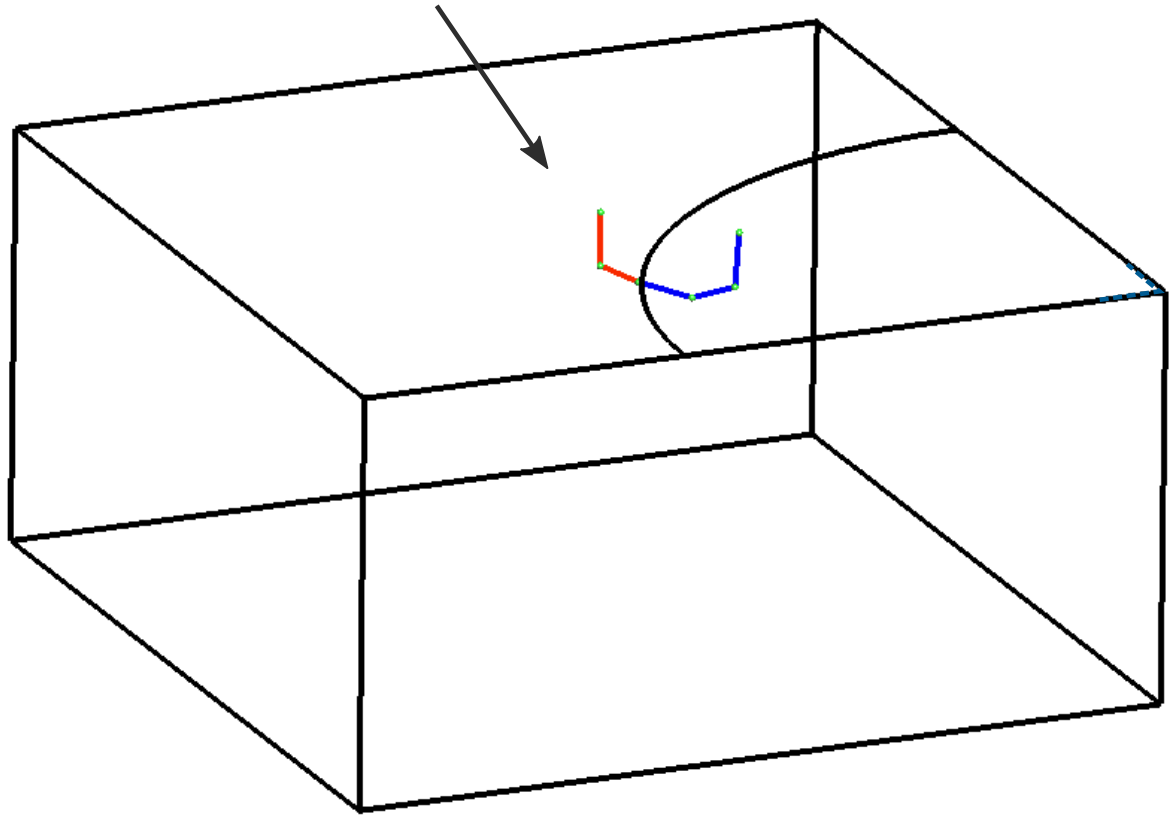
The z-axis branch is constant in the whole volume

(singular curve not tangent with FF stable direction)

See also Analysis of Non-Meshable Automatically Generated Frame Fields Viertel, Staten, Ledoux, 2016

Feature curves in 3D: only the boundary is split

”3-5 singular curve”



The z-axis branch is constant in the whole volume

(singular curve not tangent with FF stable direction)

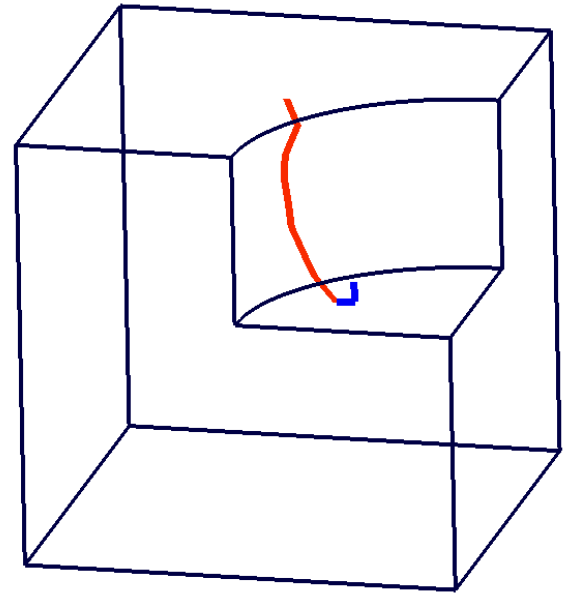
- minimize the Dirichlet energy
  (finite due to discretization)
- consistant with Laplacian kernel
- respect the boundary conditions

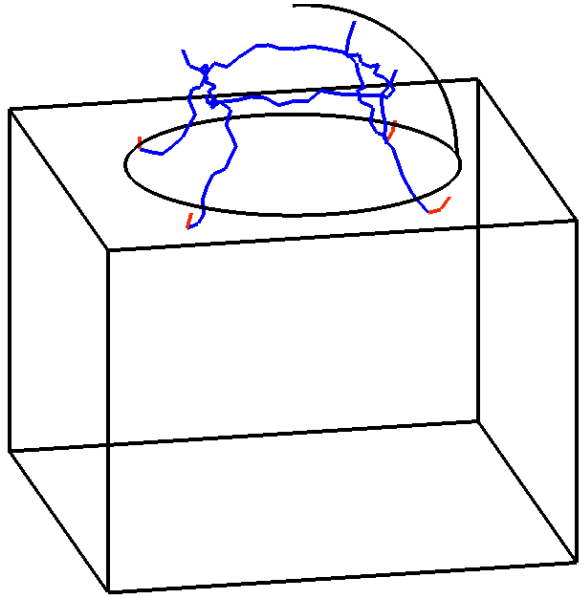
$$\min \int_\Omega ||\nabla \mathbf{f}||^2$$

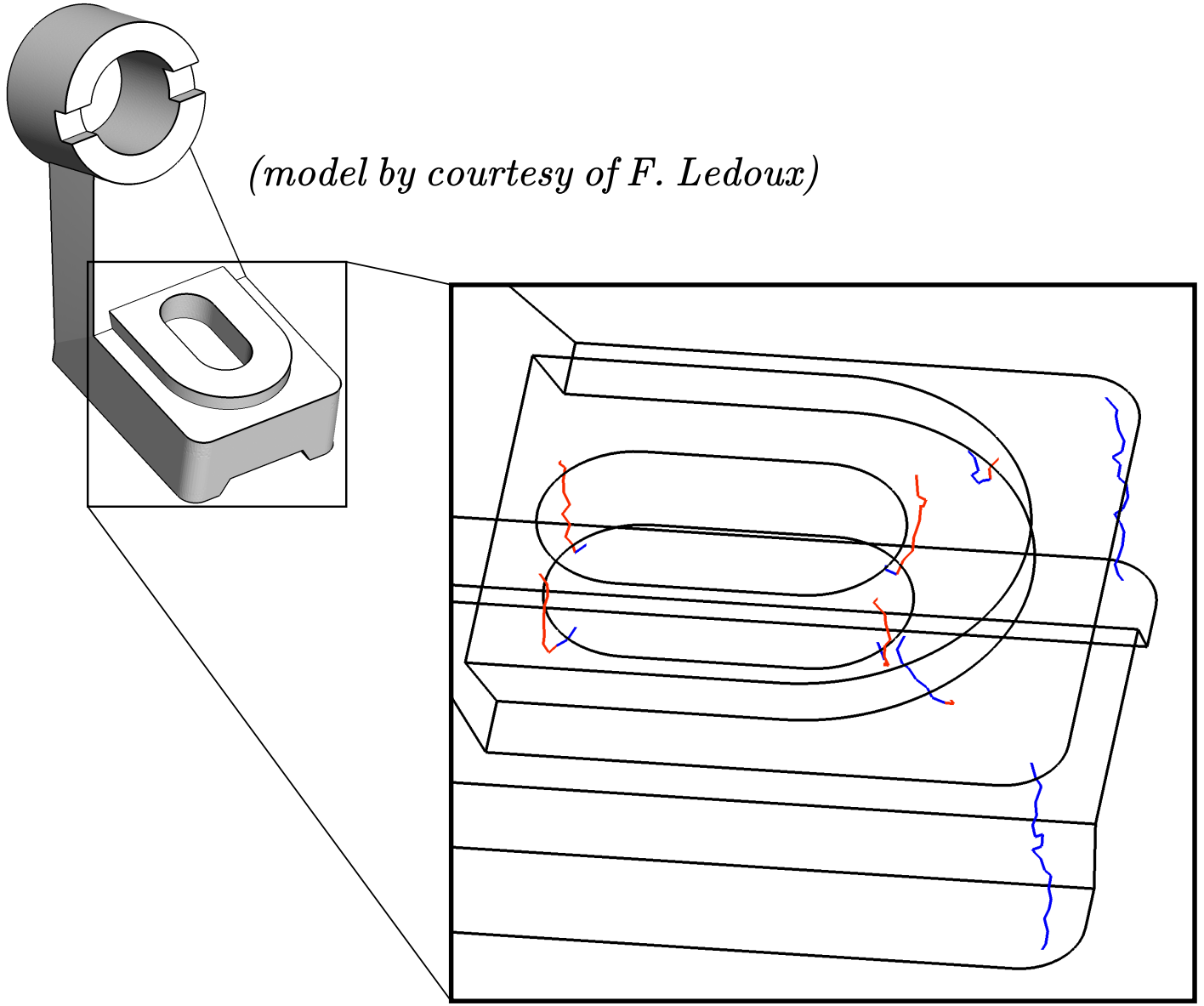
$$\mathbf{f} \;//\; \mathbf{n} \text{ on } \partial\Omega$$

Same issue with all existing frame-field solvers

Feature curves in 3D: only the boundary is split







(model by courtesy of F. Ledoux)

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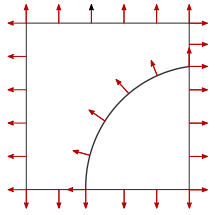

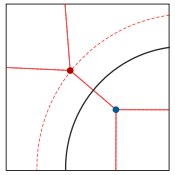

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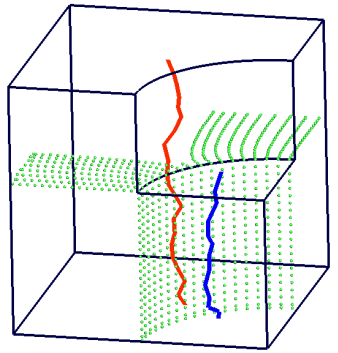

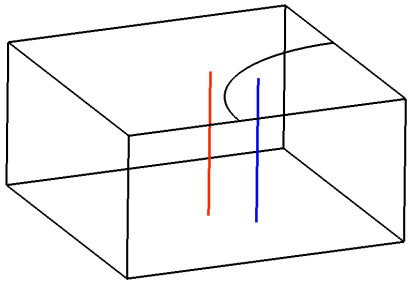

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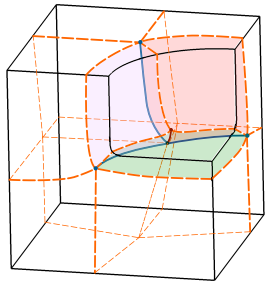

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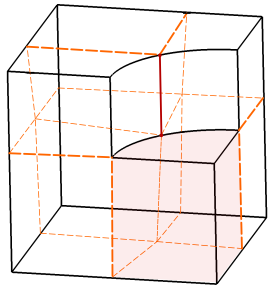
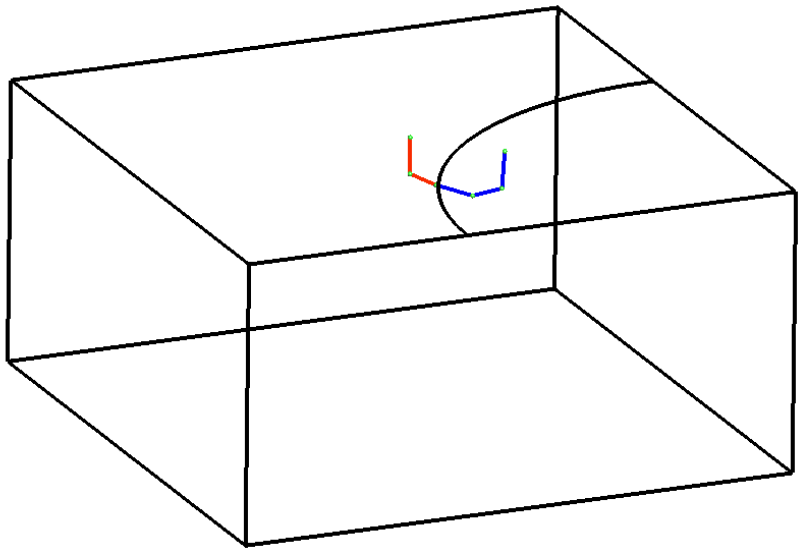
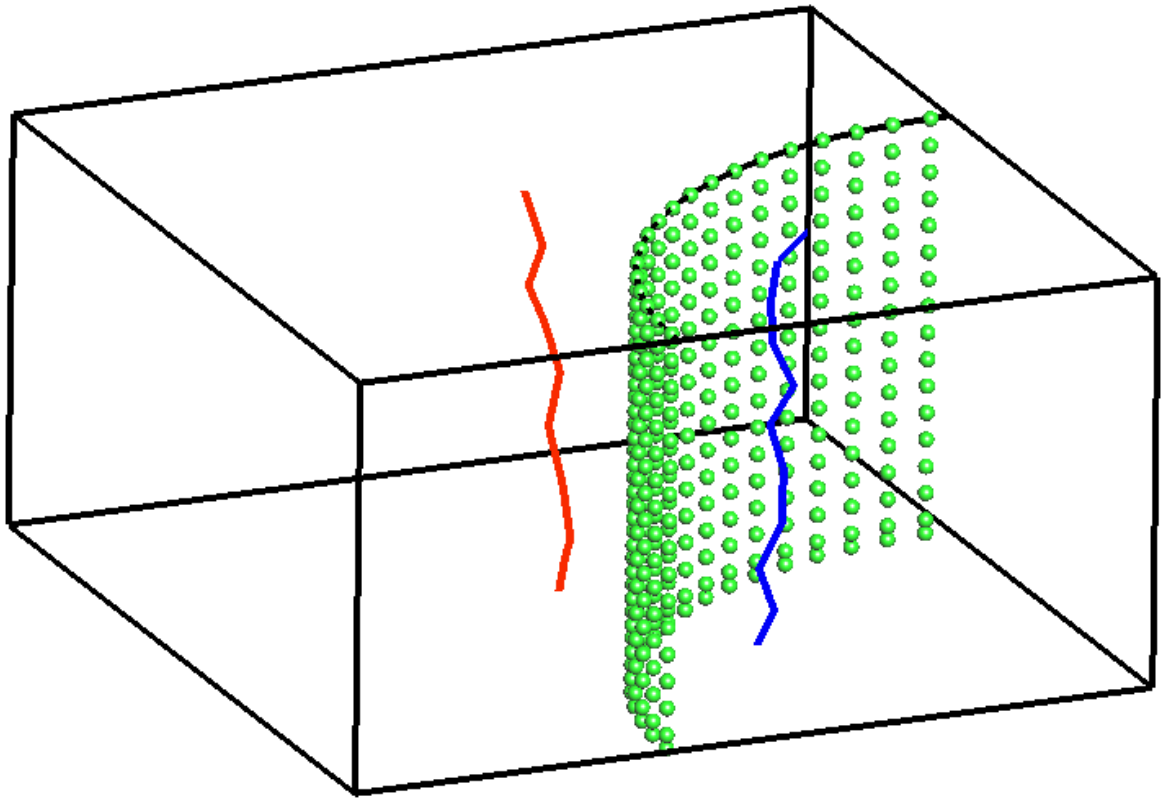


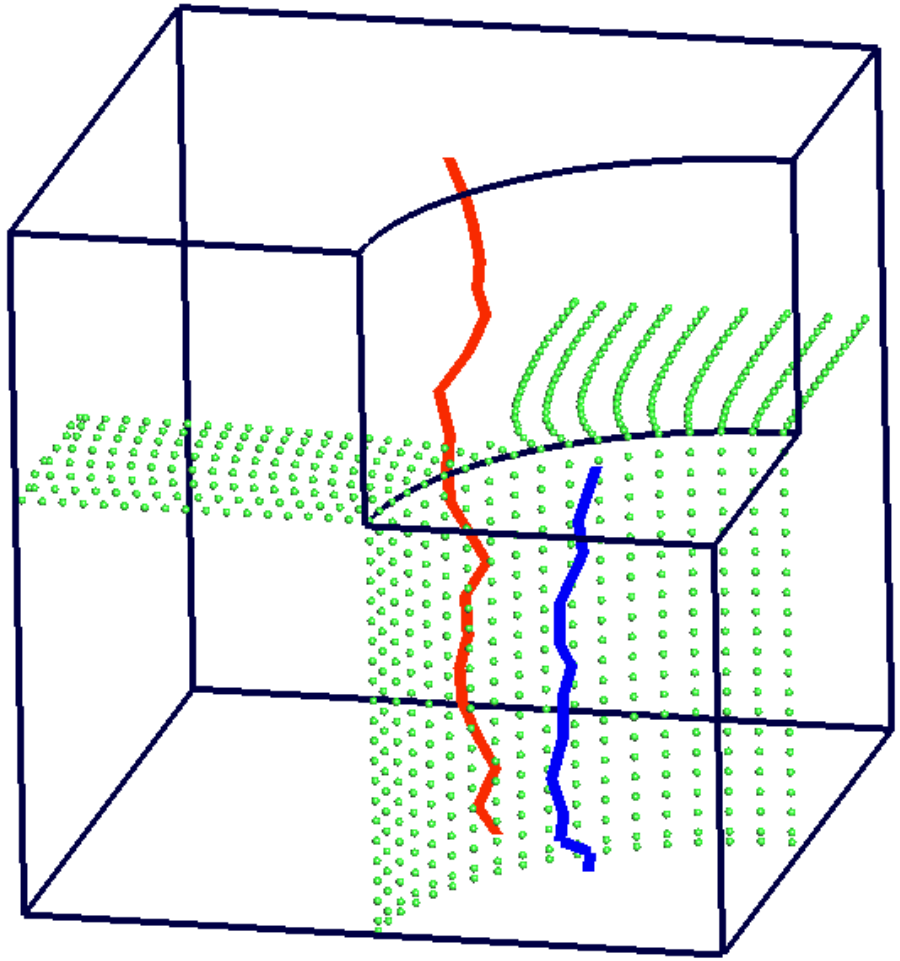
Frame field correction: volume splitting via extrusion of feature curve

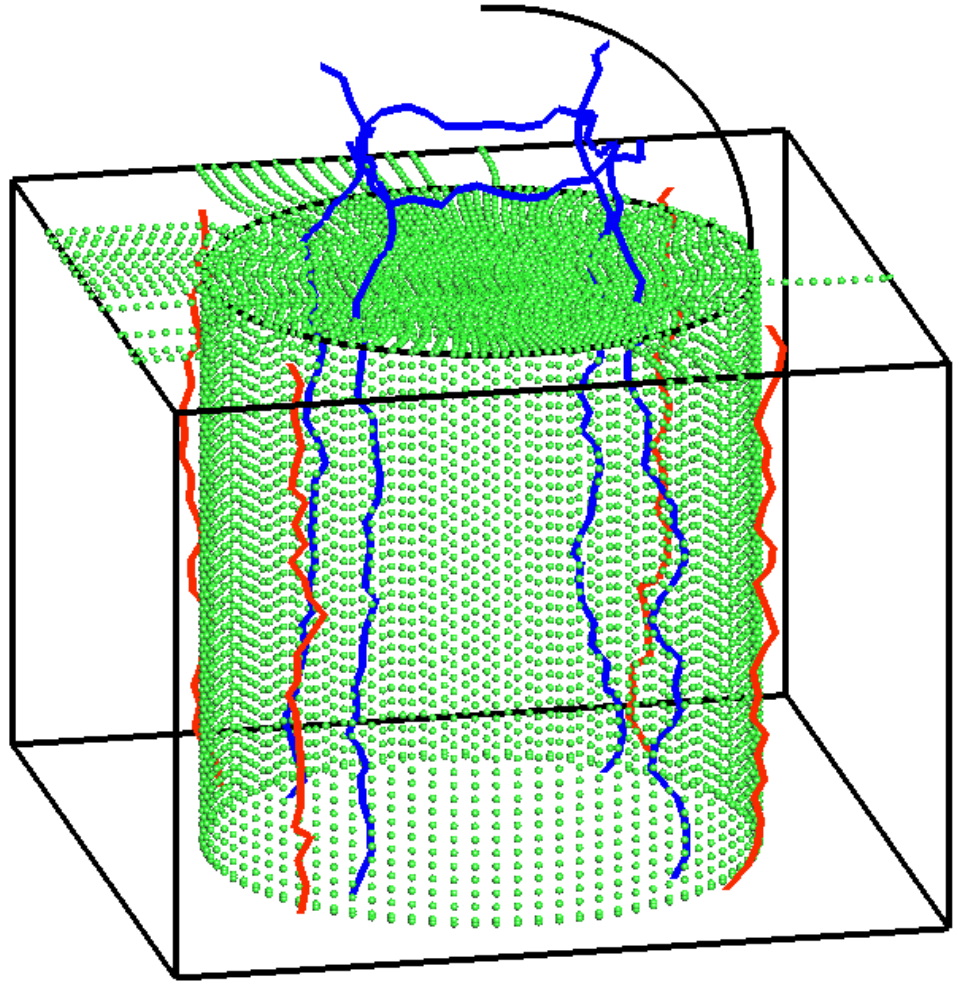




1. Trace streamlines from concave and curved feature curves (green)
2. Compute a new frame field with internal constraints
   (tangency to internal surfaces made of streamlines)

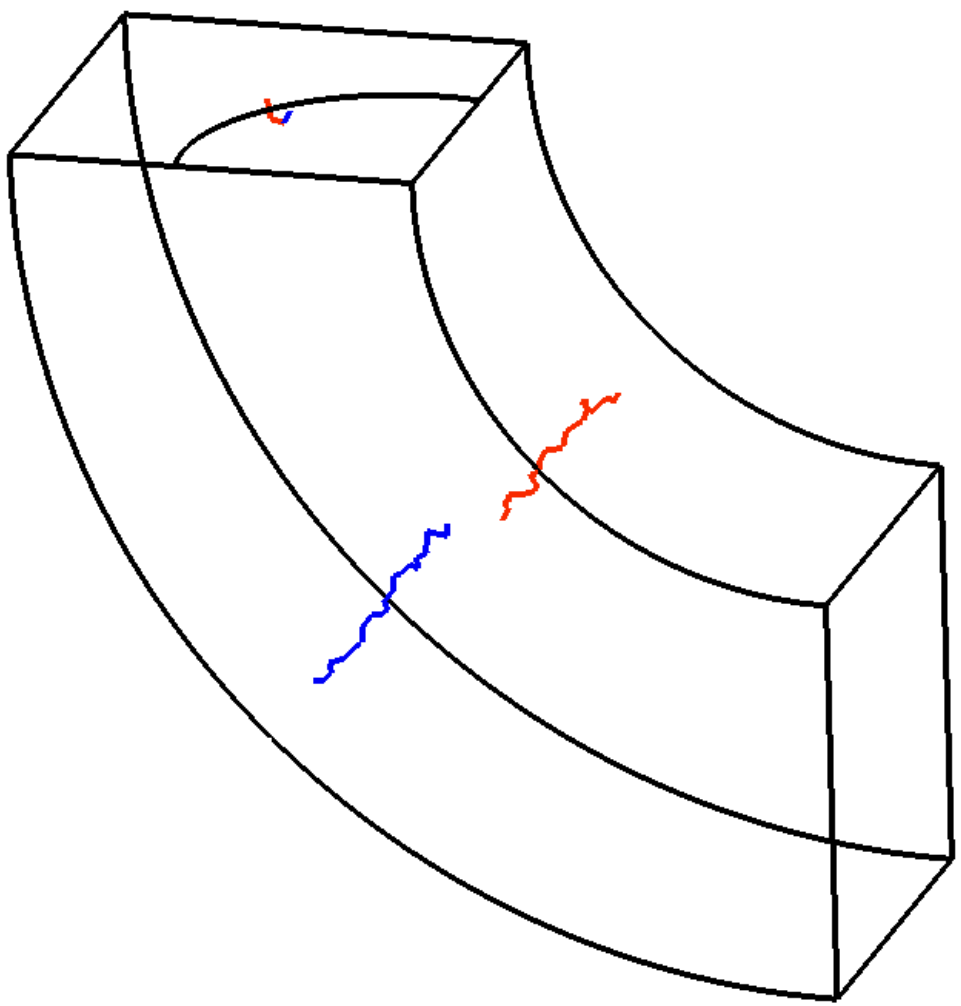
Frame field correction: extrusion of feature curve, examples

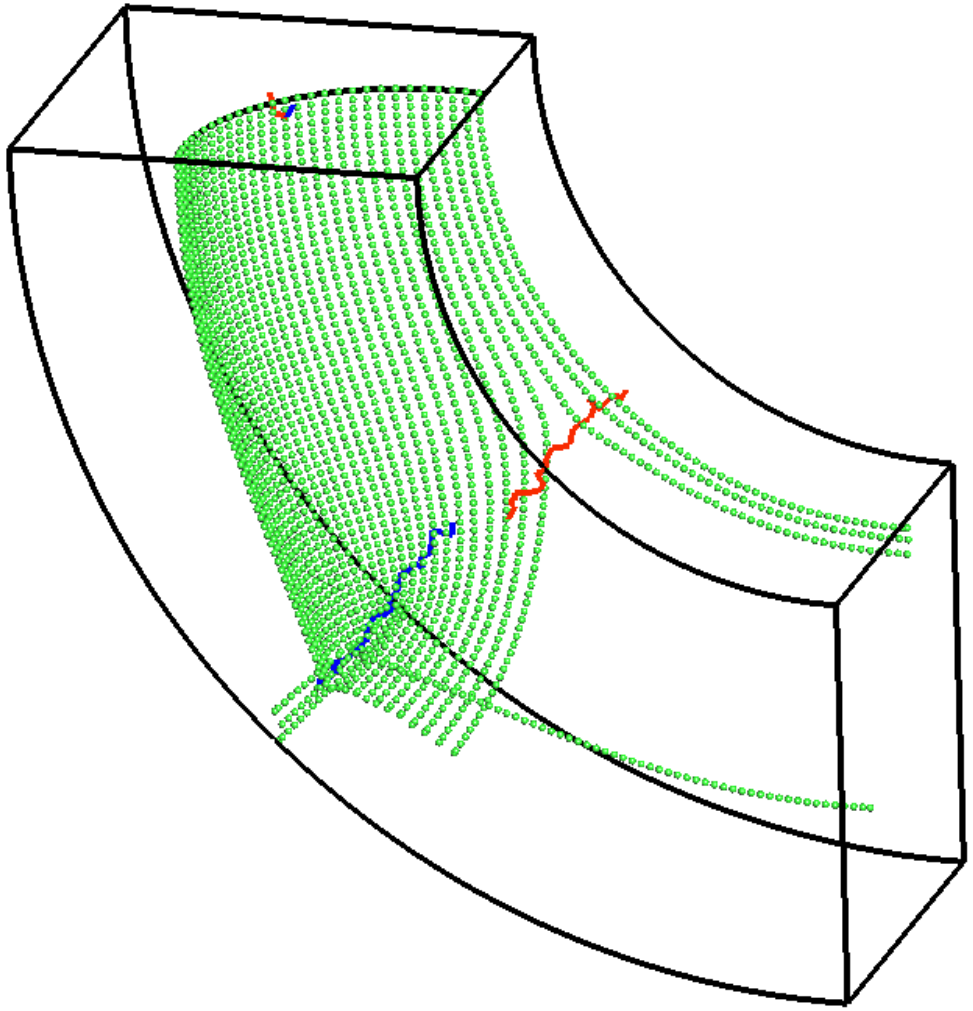




Internal constraints prevent the merge of val. 3 and val. 5 singularities

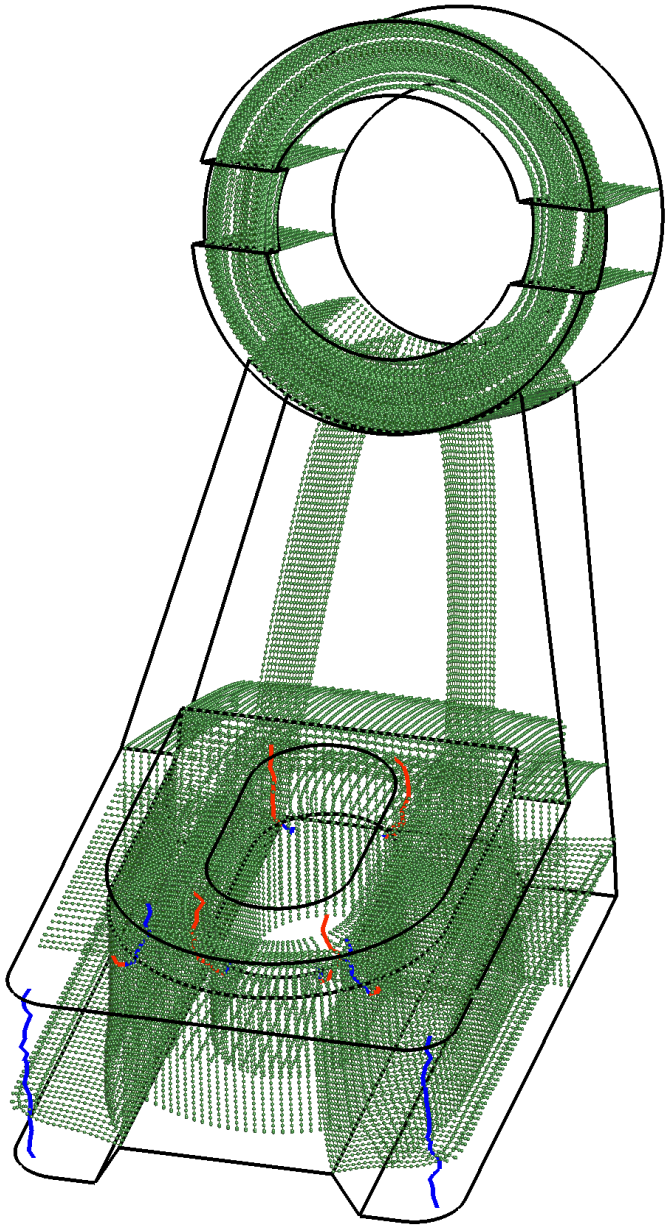
Frame field correction: extrusion of feature curve, failure case





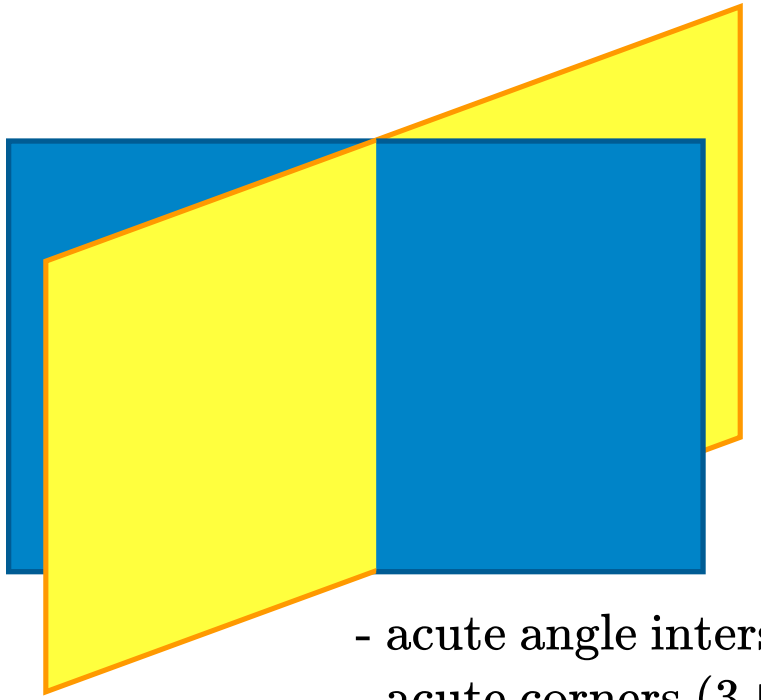
Interaction with other singularities,
extruded surfaces torn in multipe directions

Frame field correction: extrusion of feature curve



On ”non trivial” models :

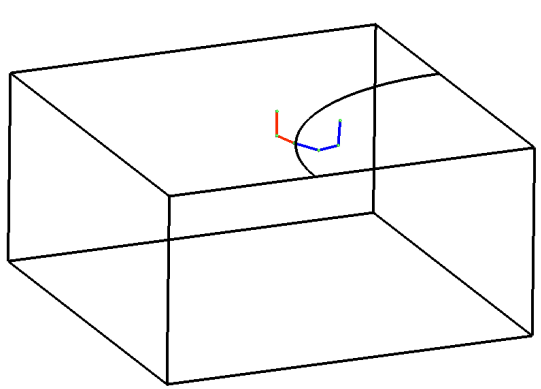
- lot of non orthogonal surface intersections
- hard to compute a new frame field because of BCs

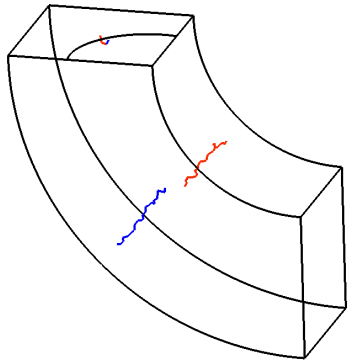


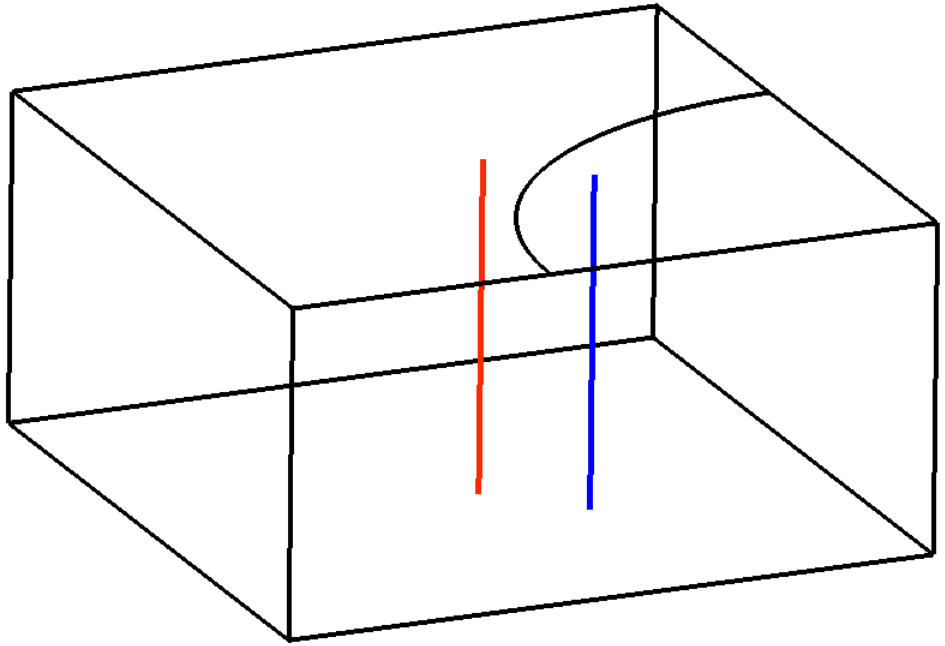
- acute angle intersection
- acute corners (3+ surfaces)
- etc

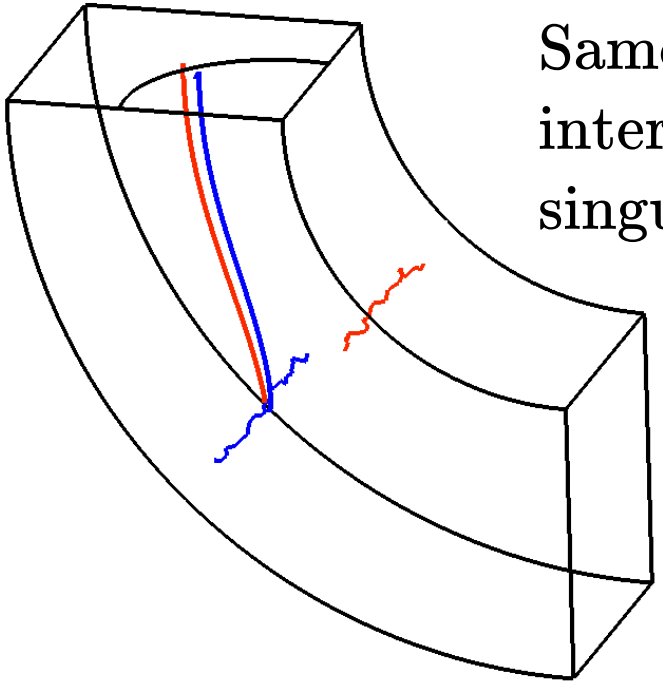
Frame field correction: extrusion of boundary singular nodes

Alternative approach: extrude singular nodes [Zheng et al. 18]



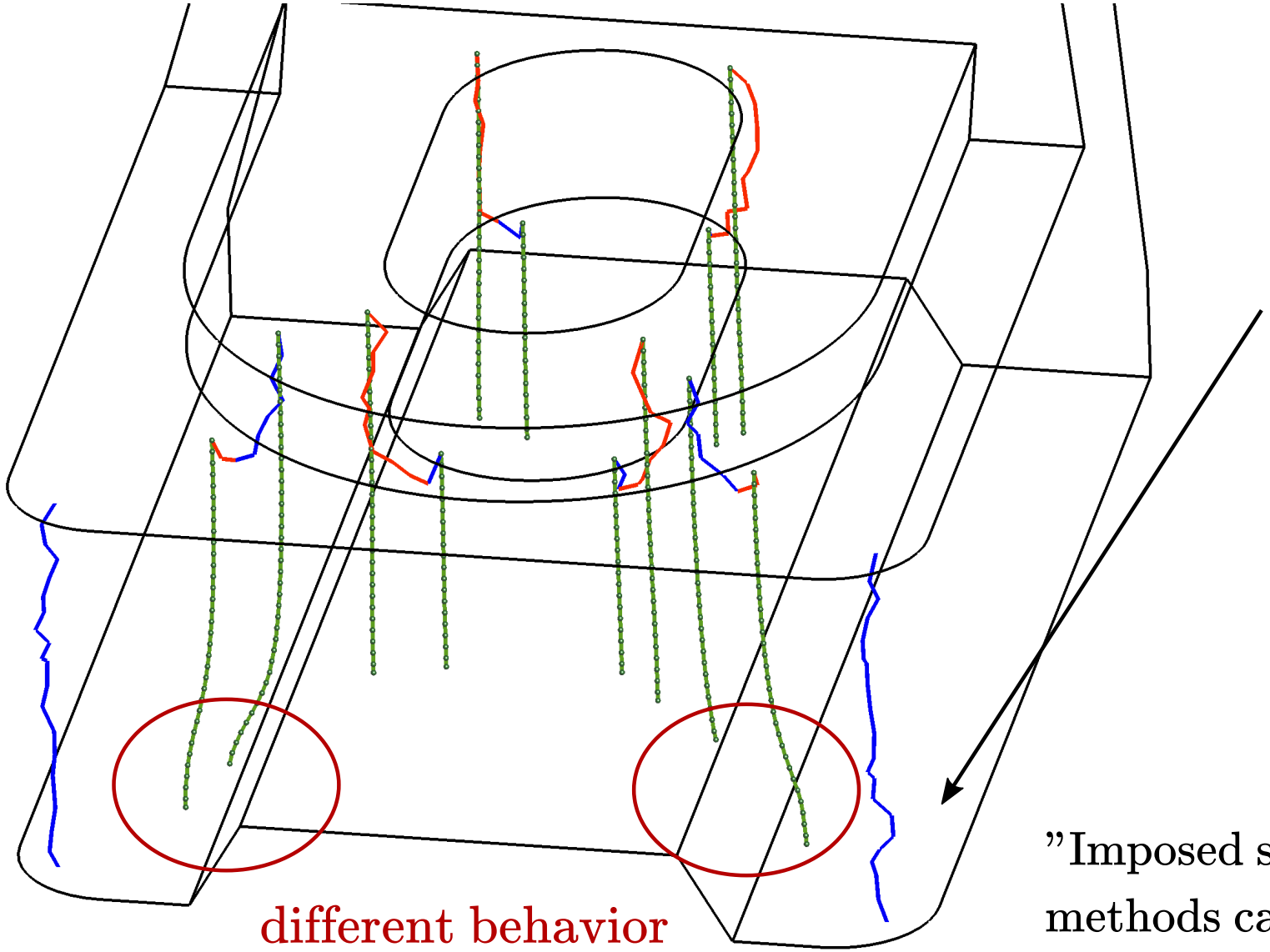






Same failure behavior: interaction with other singularities

Frame field correction: extrusion of boundary singular nodes



different behavior

Boundary patch :
- 3 sided
- one val. 3 singularity
- one val. 5 singularity

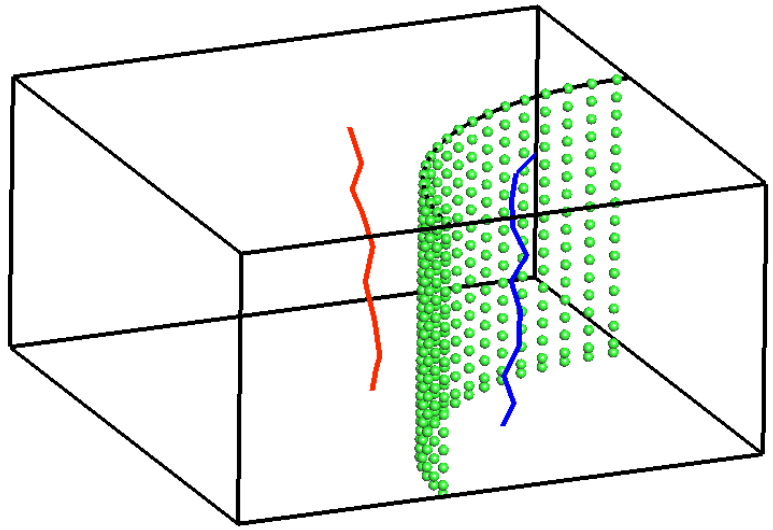
Not compatible with Poincare-Hopf theorem

”Imposed singularity graph” frame field methods cannot work with this input

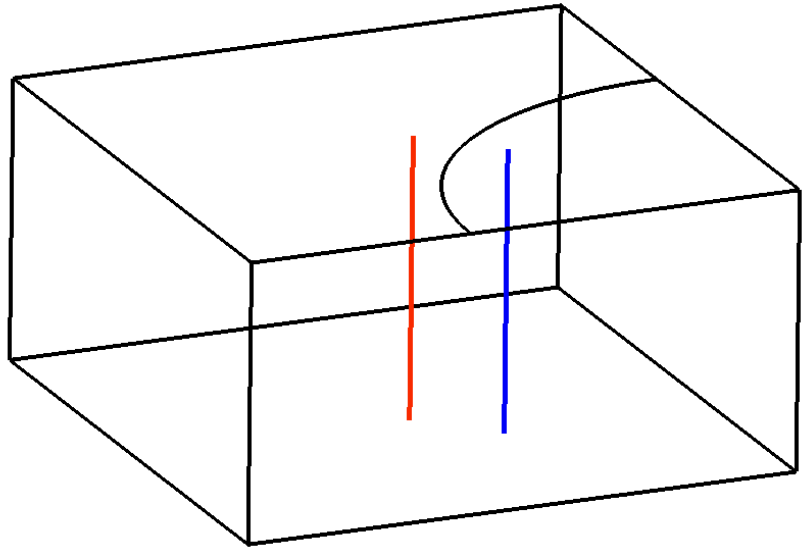
Frame field correction: conclusion on extrusion approaches

Extrusion, following frame field stable direction, of :

- feature curves



- boundary singular nodes

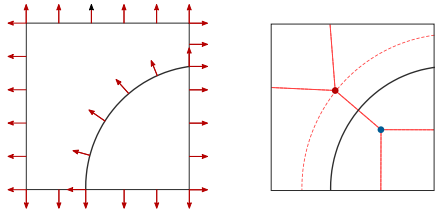


- Works on simple models (~ extruded 2D)

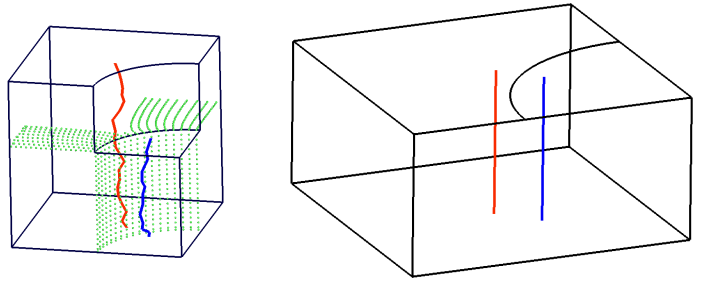
- Does not scale with model complexity
  (extrusion process can fail, new frame field cannot be computed, etc)

Overview - Multiple approaches to frame field correction

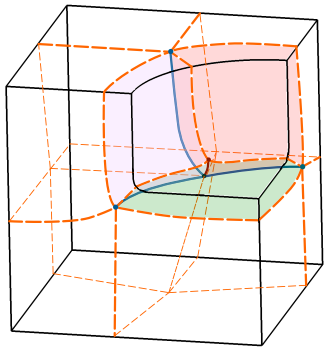
1. Issue analysis



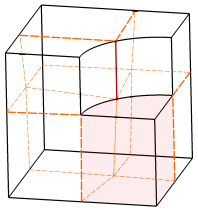
2. Extrusion of feature curves / bdr. singularilities



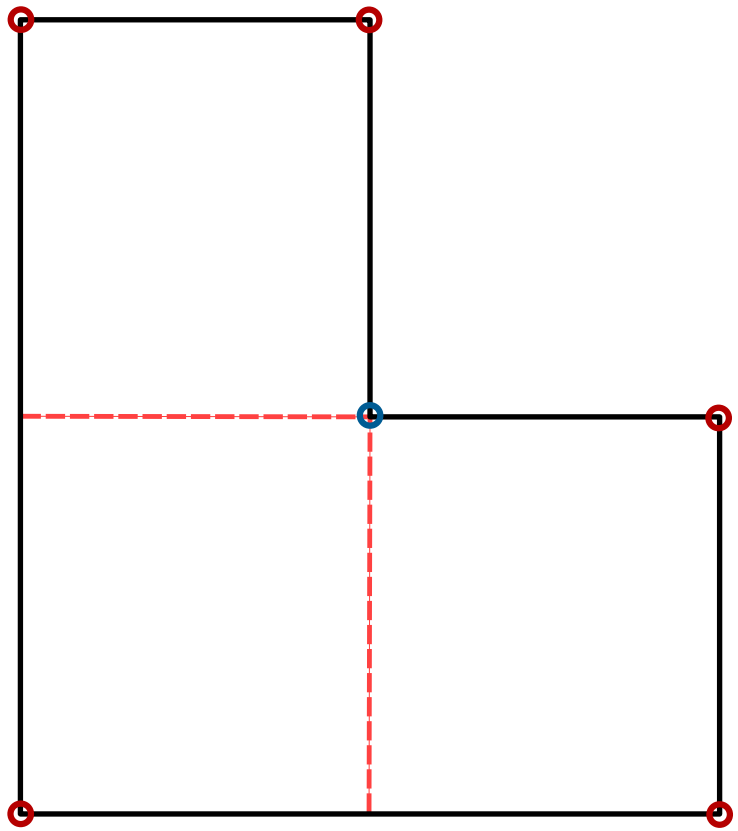
3. Smoothing of feature curves



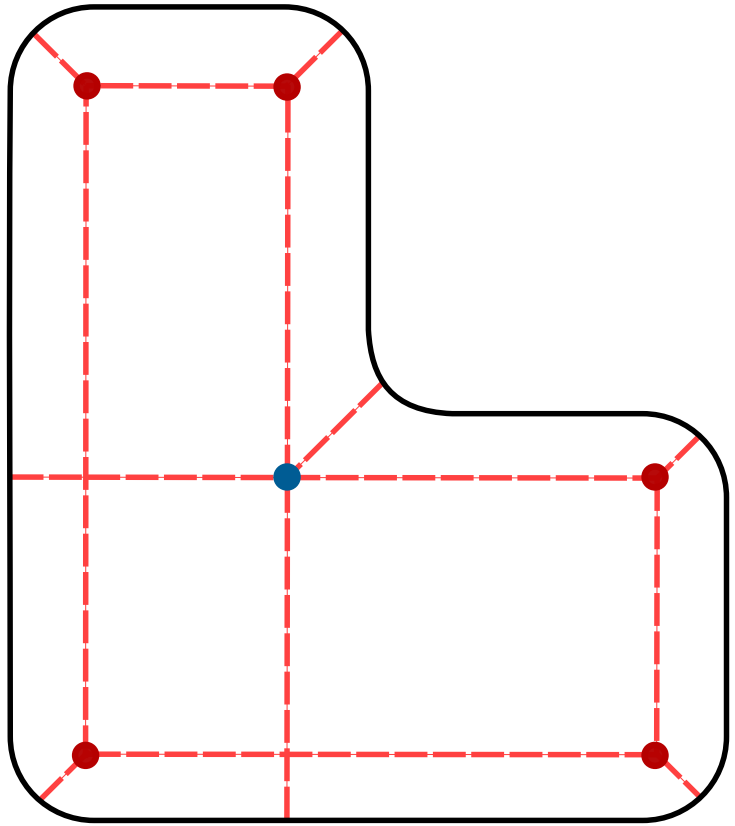
4. Invalid singular curve boundary snapping



Frame field correction: feature curve smoothing (corner in 2D)



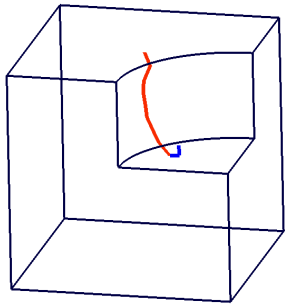
- singular corners (val. $\neq 2$)
- no interior singularities



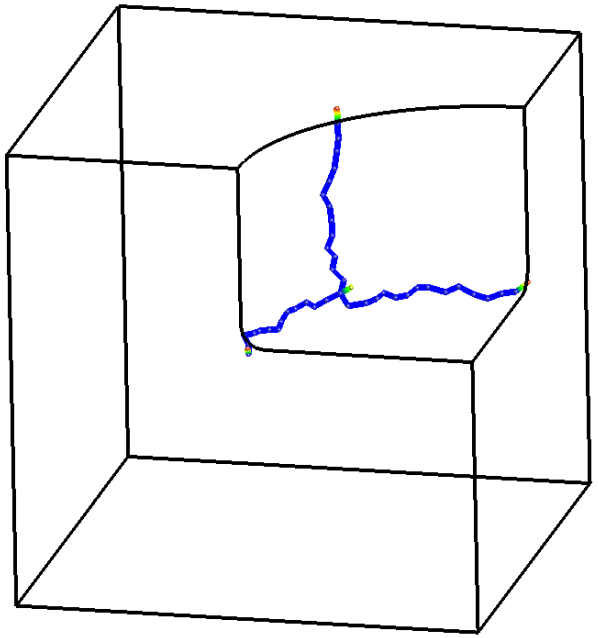
- no singular corners
- interior singularities (val. $\neq 4$)

Frame field correction: feature curve smoothing, principle

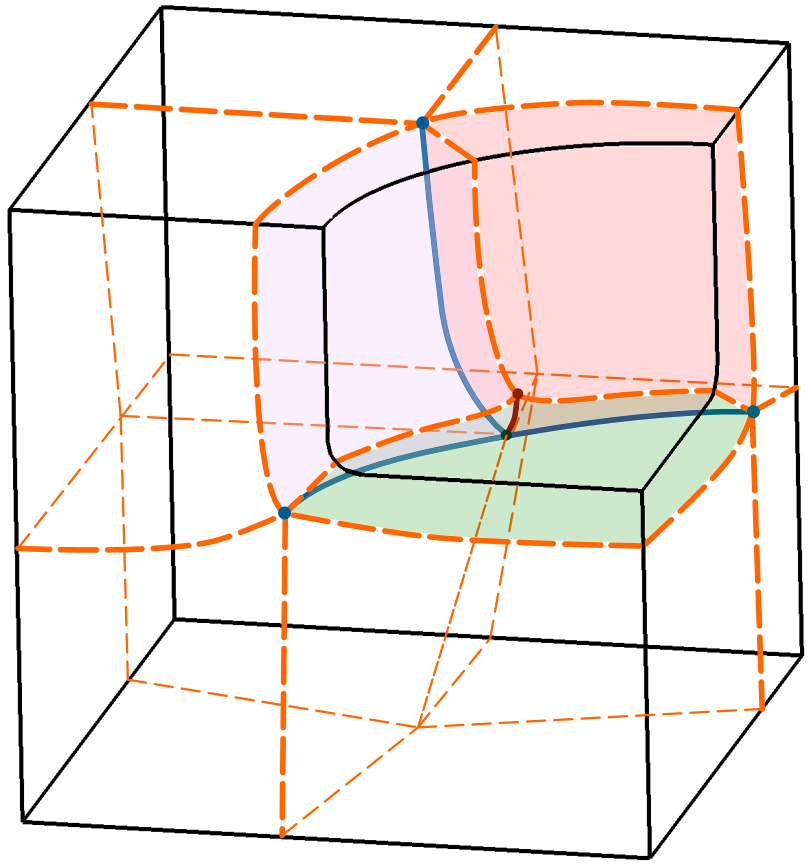
(initial, invalid graph)



(smoothed, valid graph)

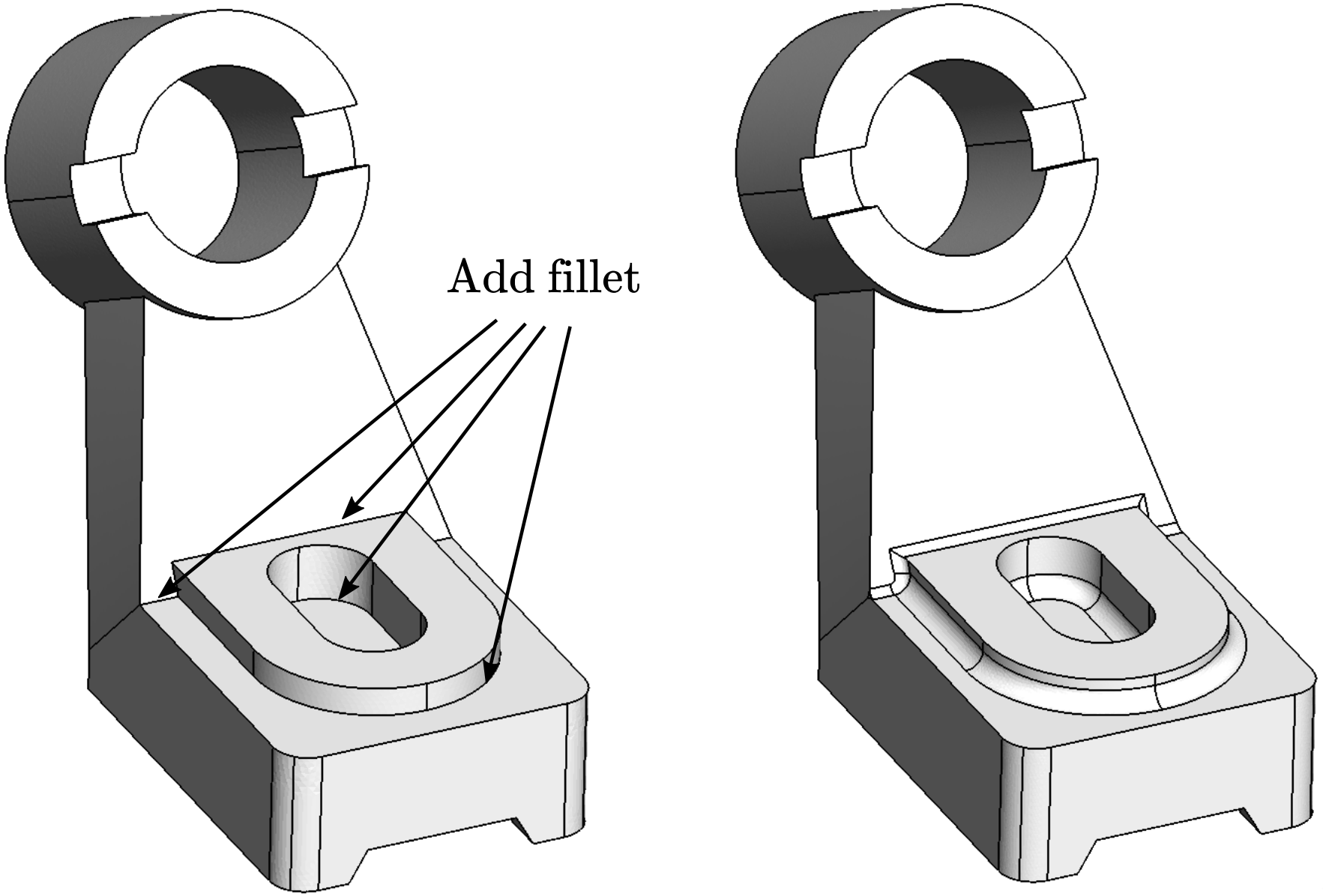


(block decomposition, by hand)

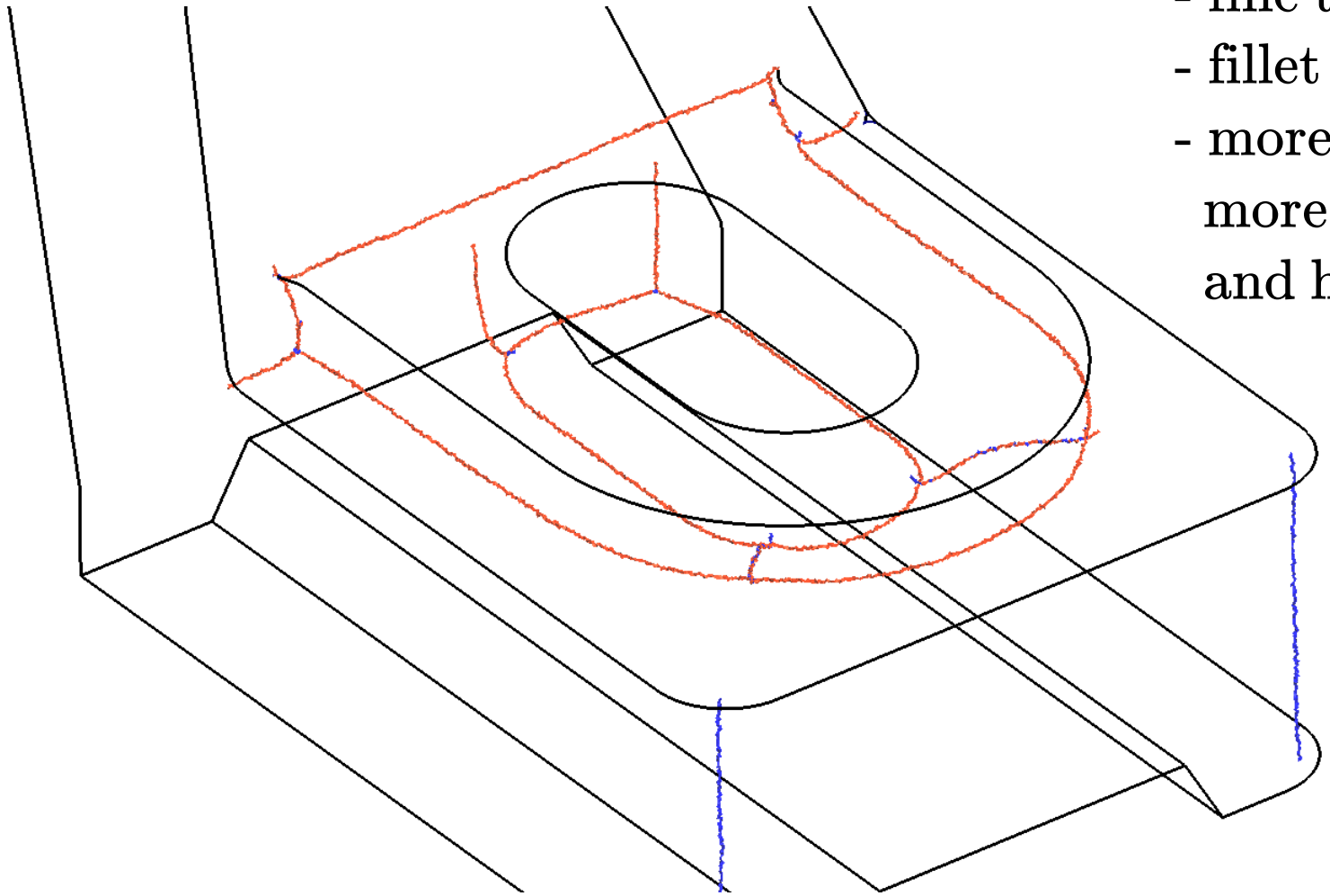


- bdr. singular node moved to fillet center
- mapping back to initial geom. is difficult
- corner with zero jacobian

Frame field correction: feature curve smoothing, CAD model



Frame field correction: feature curve smoothing, CAD model

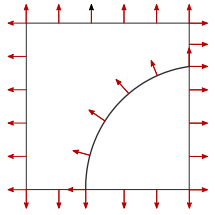


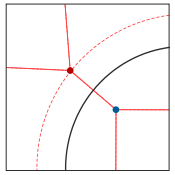
”valid” singularity graph but:

- fine tet mesh (>10M tets)
- fillet not always working (CAD)
- more complicated graph,
  more difficult to build hex mesh
  and how to map back to initial ?

Overview - Multiple approaches to frame field correction

1. Issue analysis





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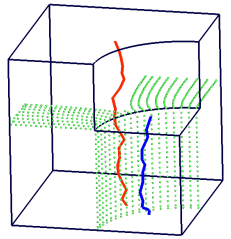
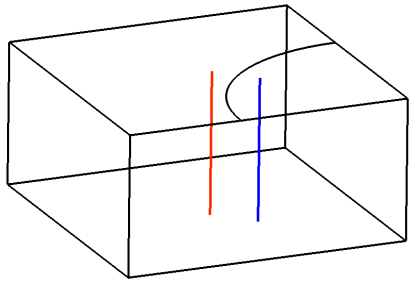
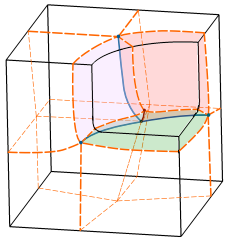
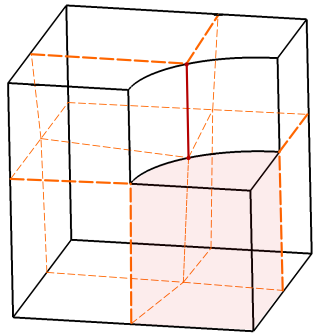




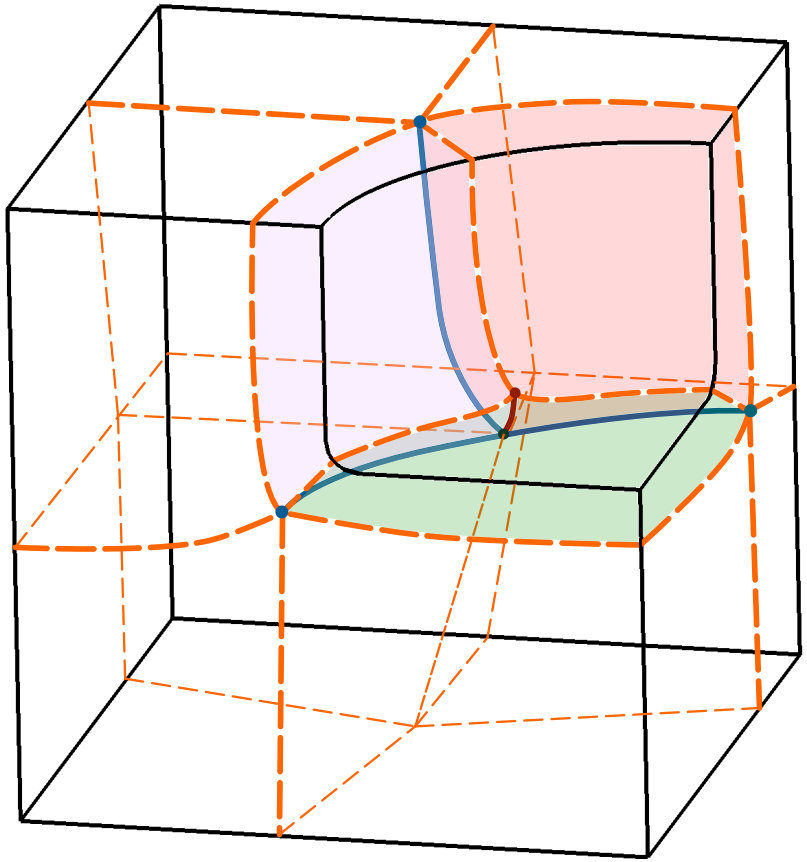
3. Smoothing of feature curves



4. Invalid singular curve boundary snapping

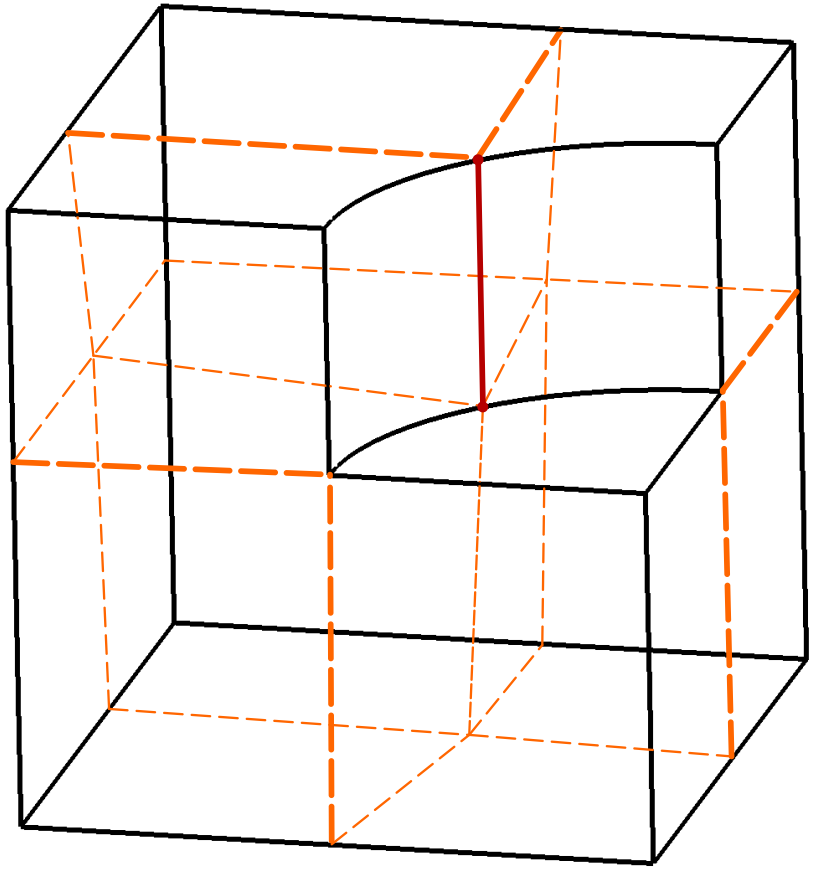


Frame field correction: boundary snapping of 3-5 sing. curves



Boundary hexahedral layer

(colored blocks)

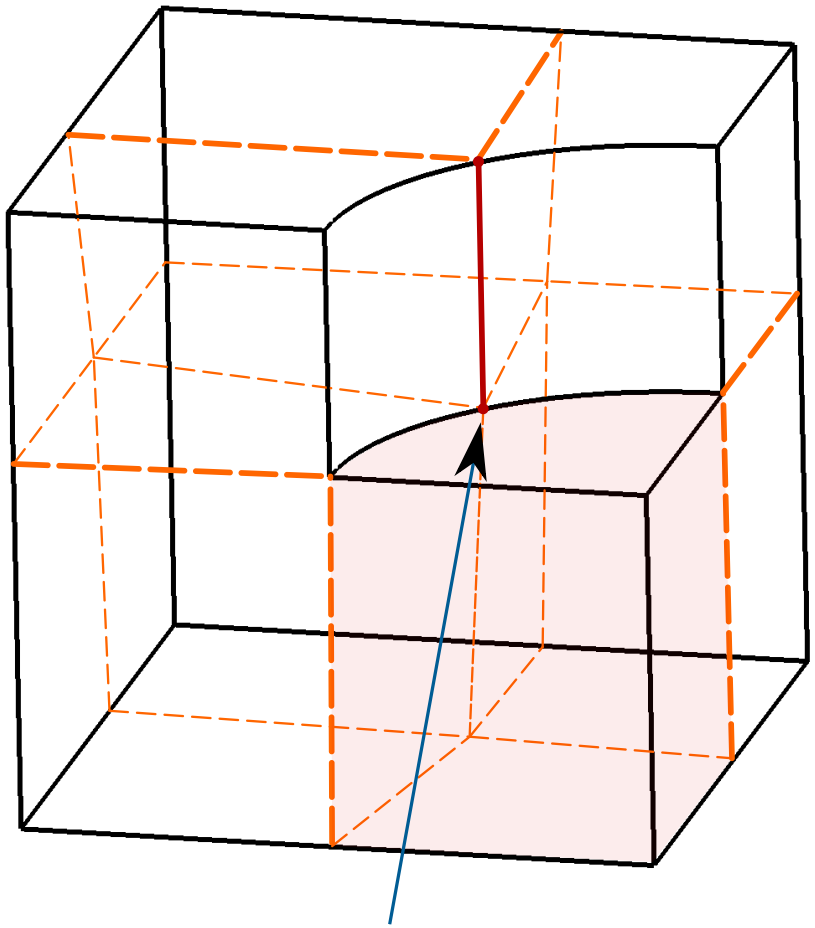


Without hex layer

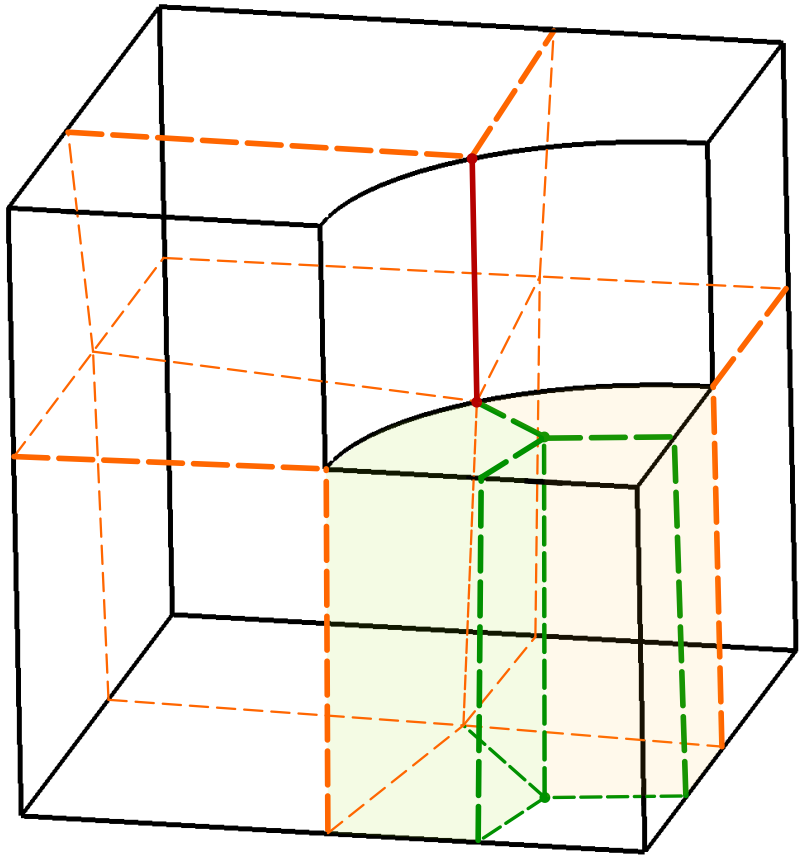
Equivalent to 3-5 sing. bdr. snapping

Valid topology but corner with zero jacobian

Valid geometry with block decomposition refinement

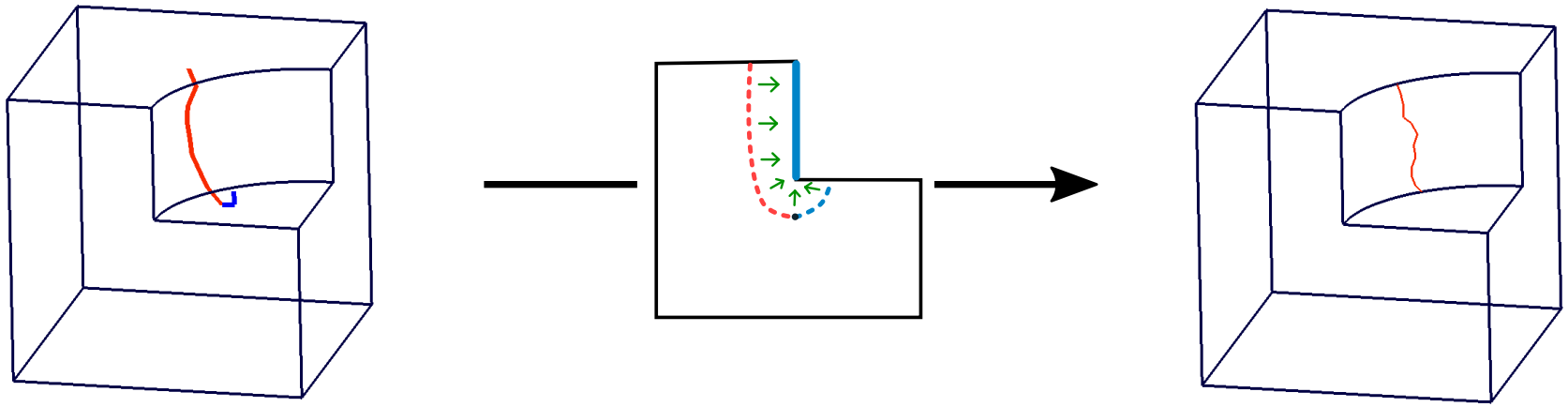


Corner with zero jacobian



After refinement of the block, valid geometry

Principle of 3-5 sing. curve boundary snapping

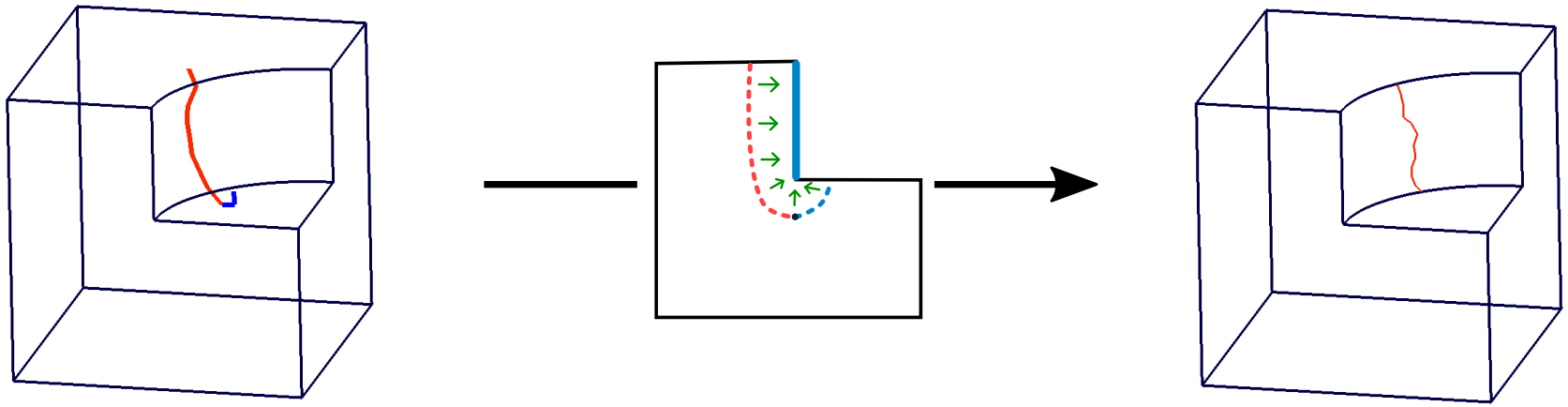


For each 3-5 singular curve:

Move bdr. node extremity to closest feature curve

Compute new boundary path between snapped extremities

Principle of 3-5 sing. curve boundary snapping



For each 3-5 singular curve:

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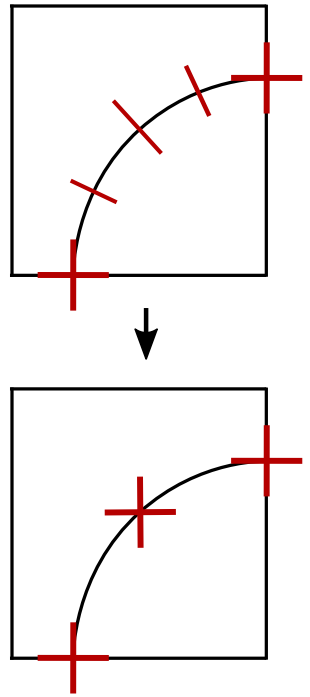


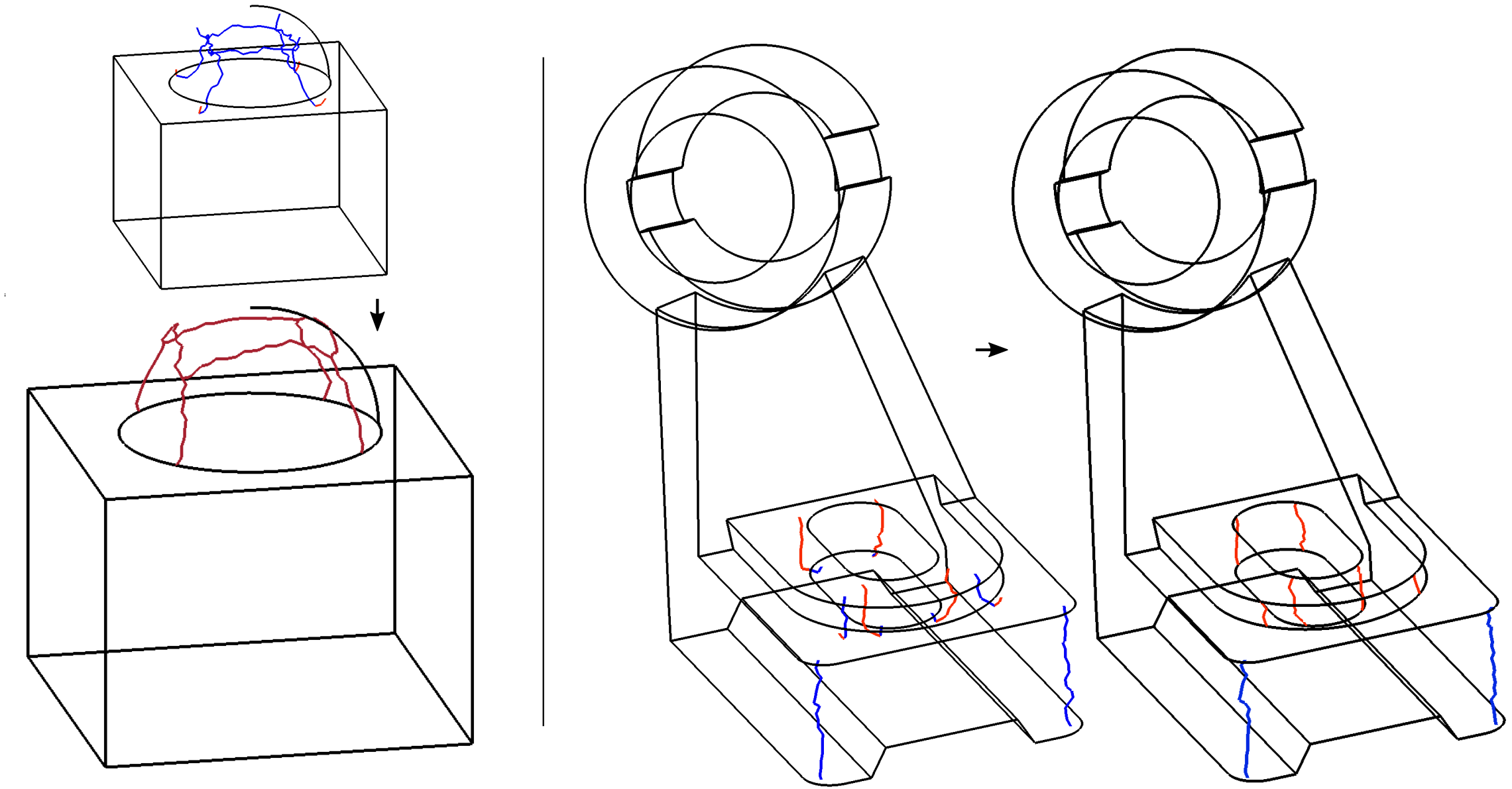
Change the frame field boundary conditions:

On the snapped curves, imposed frames are rotated by 45° (around tangent)

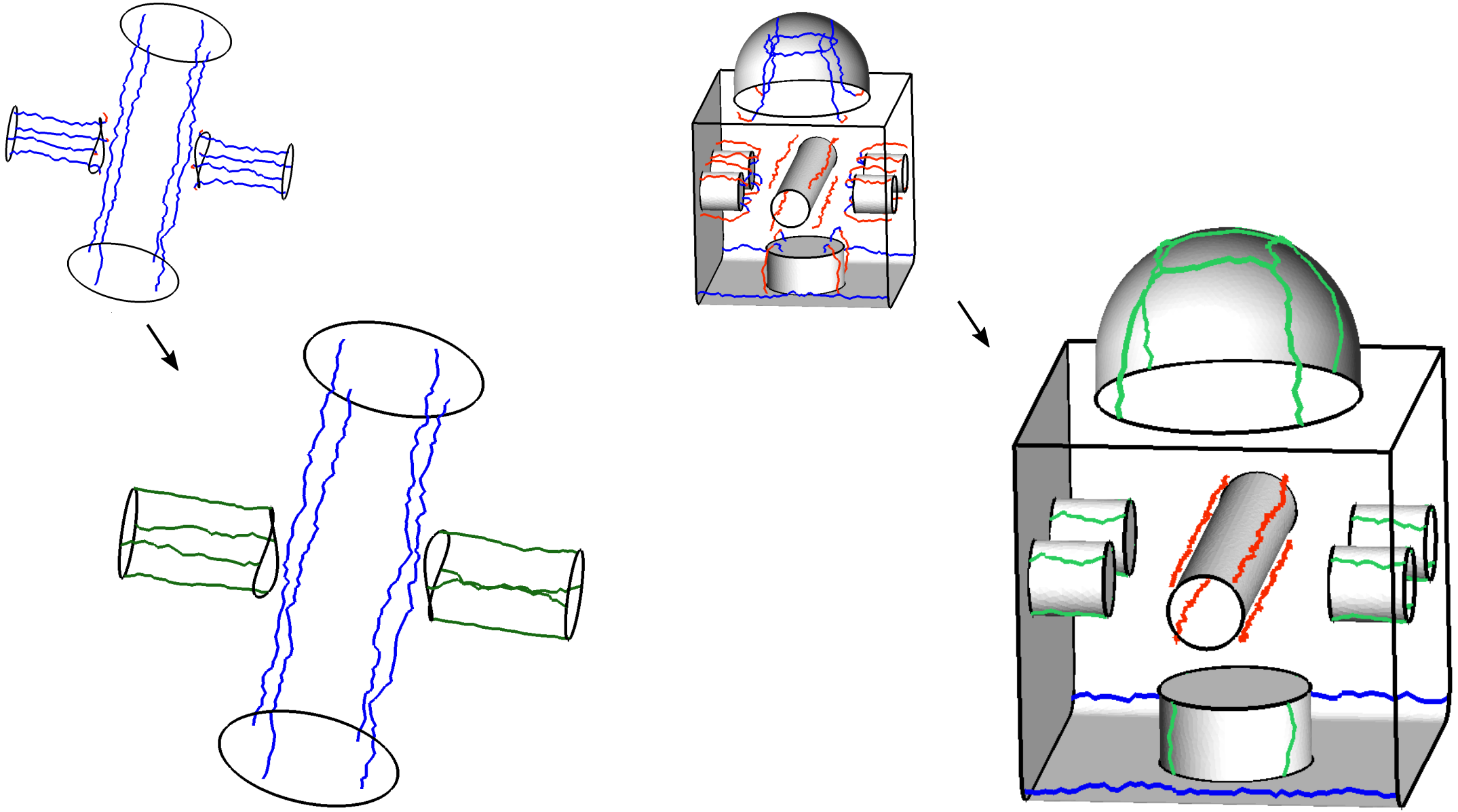
Close to snapped curves, alignement boundary conditions are removed

The new frame field is **topoligcally valid but no longer boundary-aligned**

Frame field correction, results of 3-5 sing. curve boundary snapping



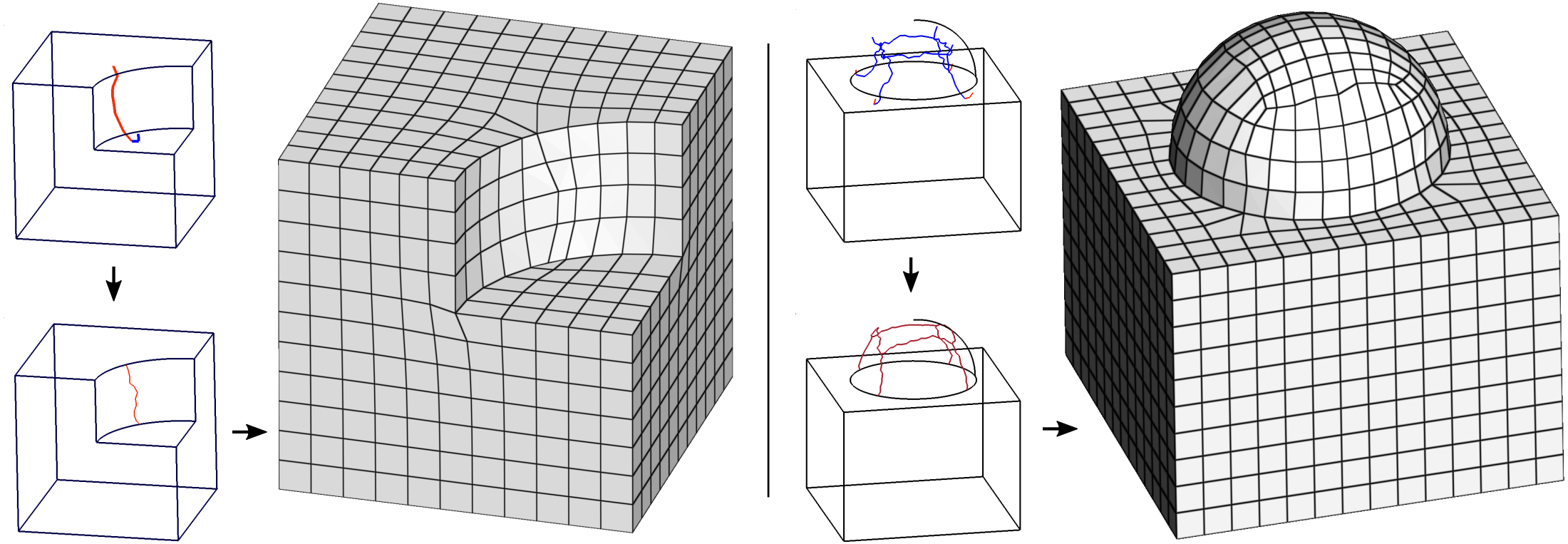
Frame field correction, results of 3-5 sing. curve boundary snapping



Frame field correction, results of 3-5 sing. curve boundary snapping

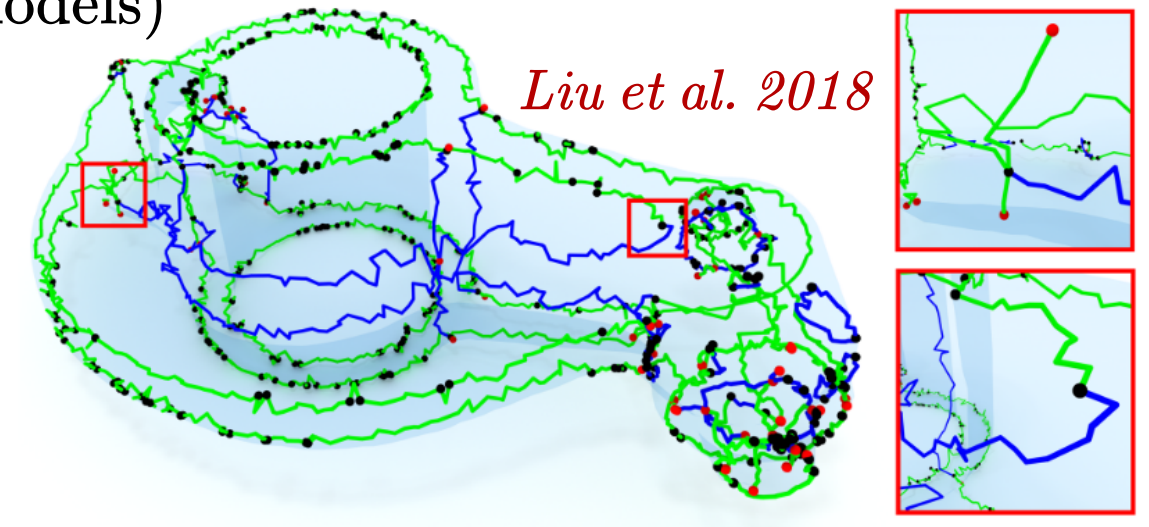
To get hexahedral meshes :

- Frame field with new BCs (changed after snapping)
- CubeCover parameterisation (using CoMISo [Bommes et al. 2011])
- Hexahedra extraction (using HexEx [Lyon et al. 2016])



Frame field correction, limitations of 3-5 sing. curve boundary snapping

- Not generic, only works when 3-5 singularities can be snapped on the boundary (but still very frequent with CAD models)

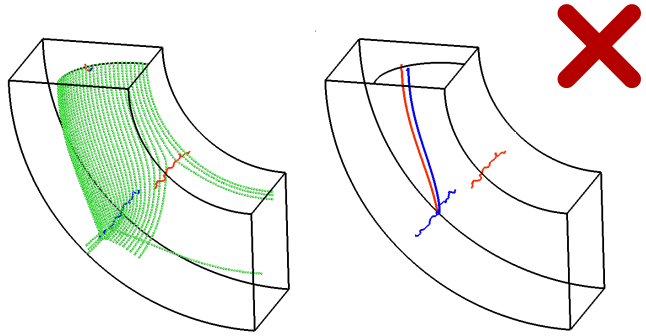


- Requires good quality frame field to extract singular curves and their valences

- Frame field is no longer boundary aligned (locally)

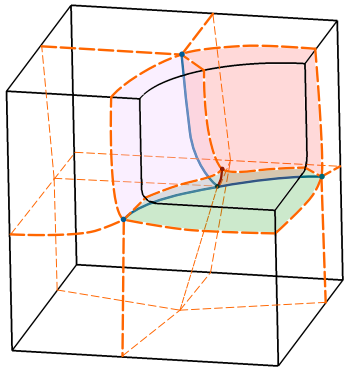
- CubeCover parametrization highly distorted

Conclusion on multiple approaches to frame field correction



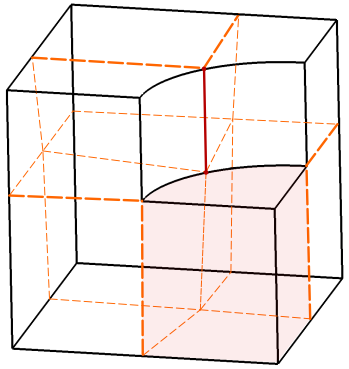
Extrusion (curve or bdr. sing.) not reliable

attempt of global correction



Smoothing / fillet : valid frame field but hard to use

local correction by adding singular curves



3-5 singularity snapping : topologically valid frame field

local correction by removing singular curves

no longer boundary aligned

require block refinement

Thank you for your attention

Any questions ?