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Towards automatic block decomposition of general 3D domains using cross fields

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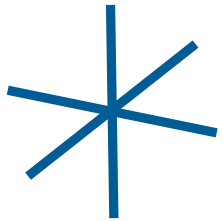
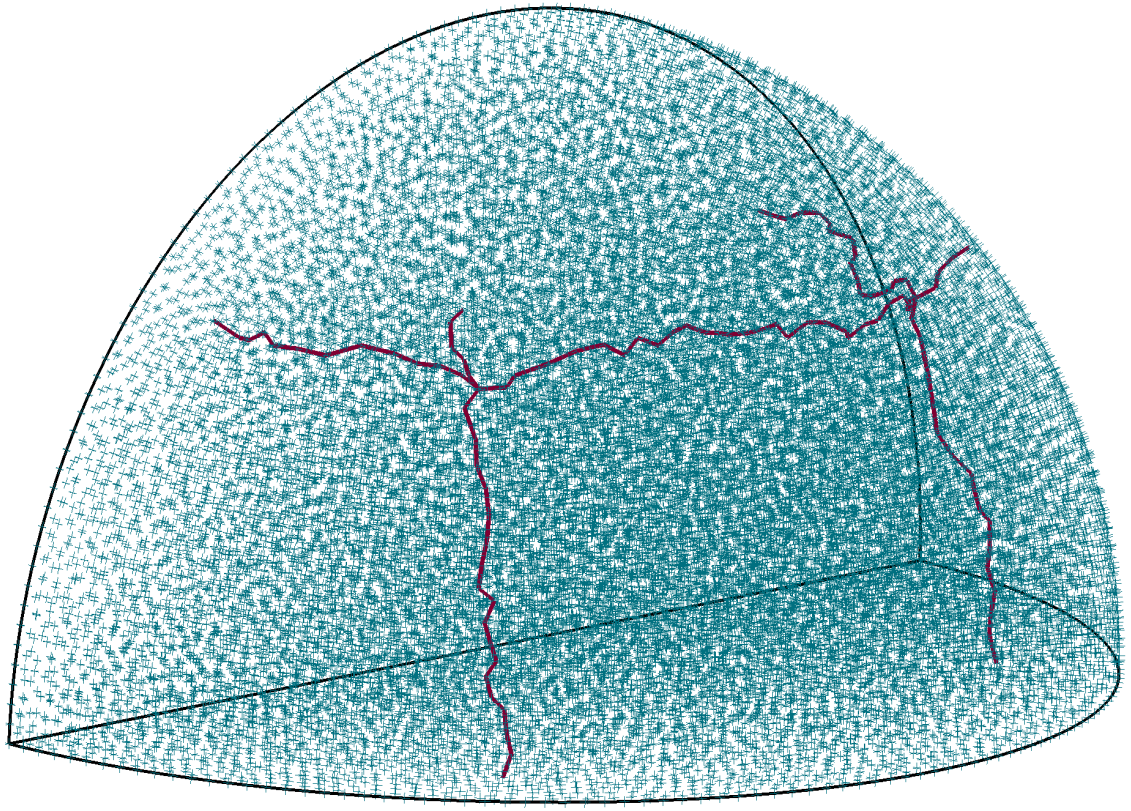
Boundary-aligned frame fields

Idea: valid frame field contains the **block decomposition topology**

2D cross:

3D cross or frame:





Frame field singularity graph: singular edges of the block decomposition
(Kowalski et al. 2014, etc)

Boundary-aligned frame fields

(1) Minimize the Dirichlet energy

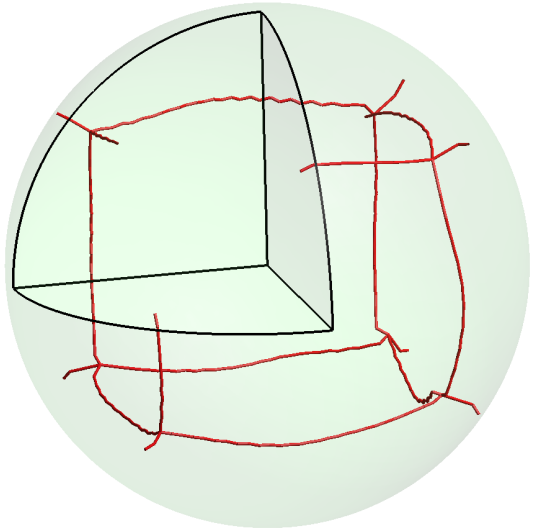
$$\min \int_\Omega ||\nabla \mathbf{f}||^2$$

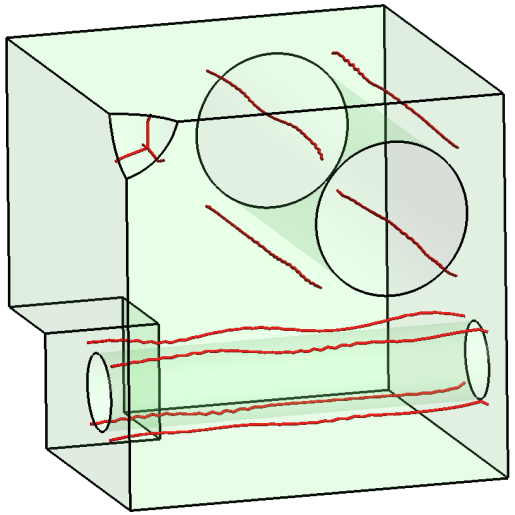
(2) Boundary alignement (one direction imposed)

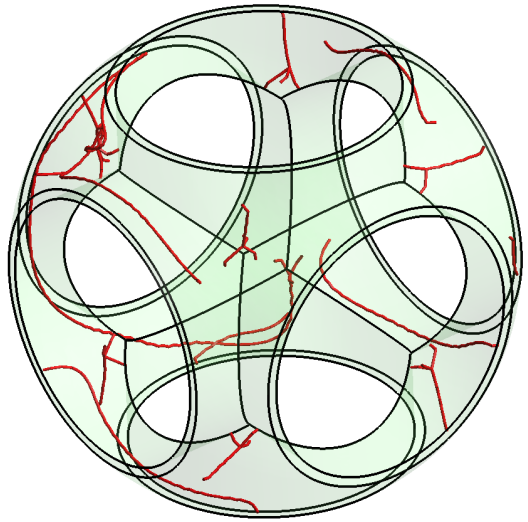
$$\mathbf{f} \perp \mathbf{n} \text{ on } \partial\Omega$$

(3) Imposed frames on feature curves and corners, directions given by adjacent surface normals

$$\mathbf{f} = \mathbf{b} \text{ on } \partial\Omega_C$$







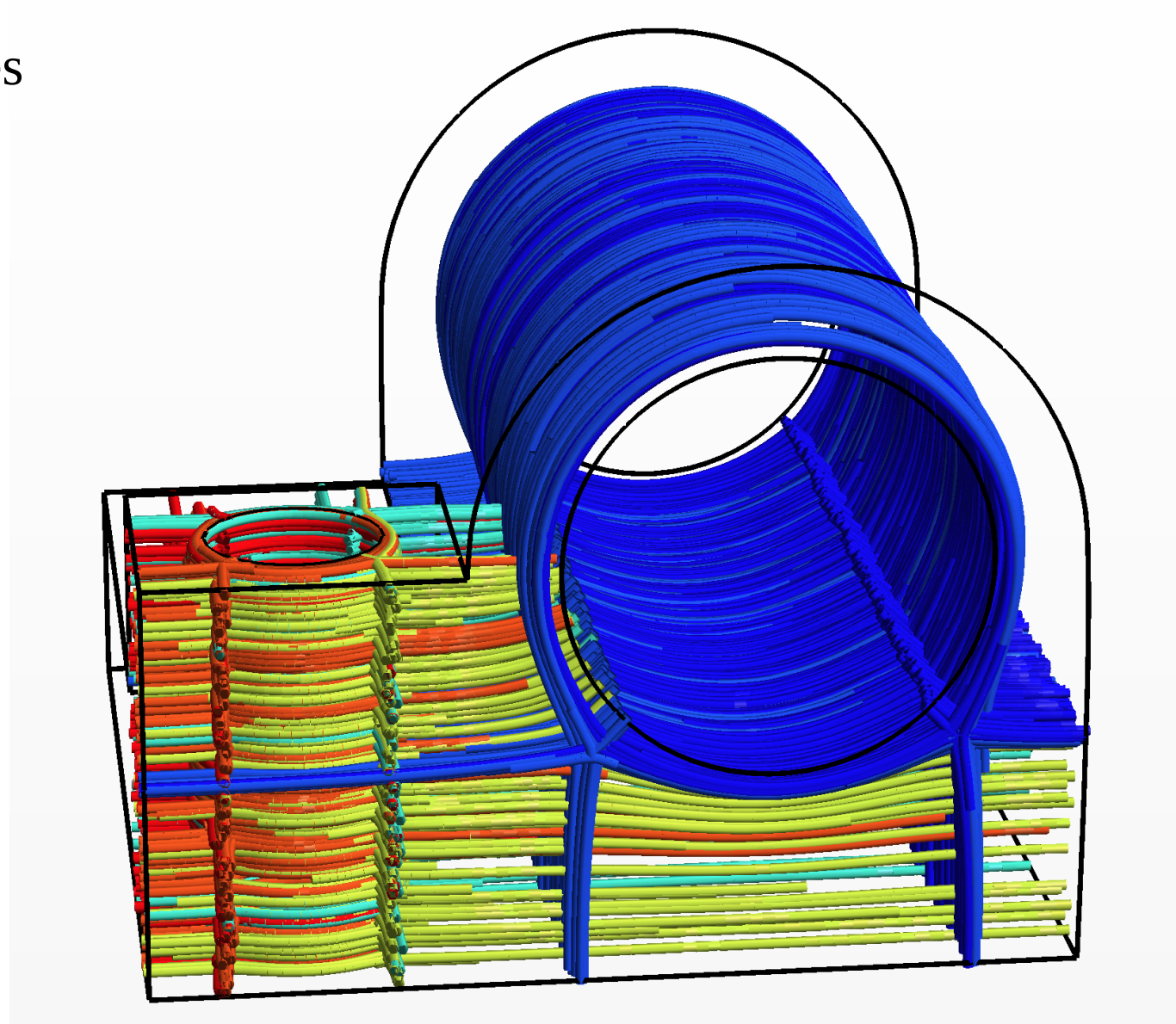
References:

Huang et al. 2011,
Li et al. 2012,
Ray et al. 2016,
Solomon et al. 2017.
Chemin et al. 2018,
and others

Selected streamlines in boundary-aligned 3D frame field

Streamlines from singular curves (tracing with RK4)

The block decomposition is well visible



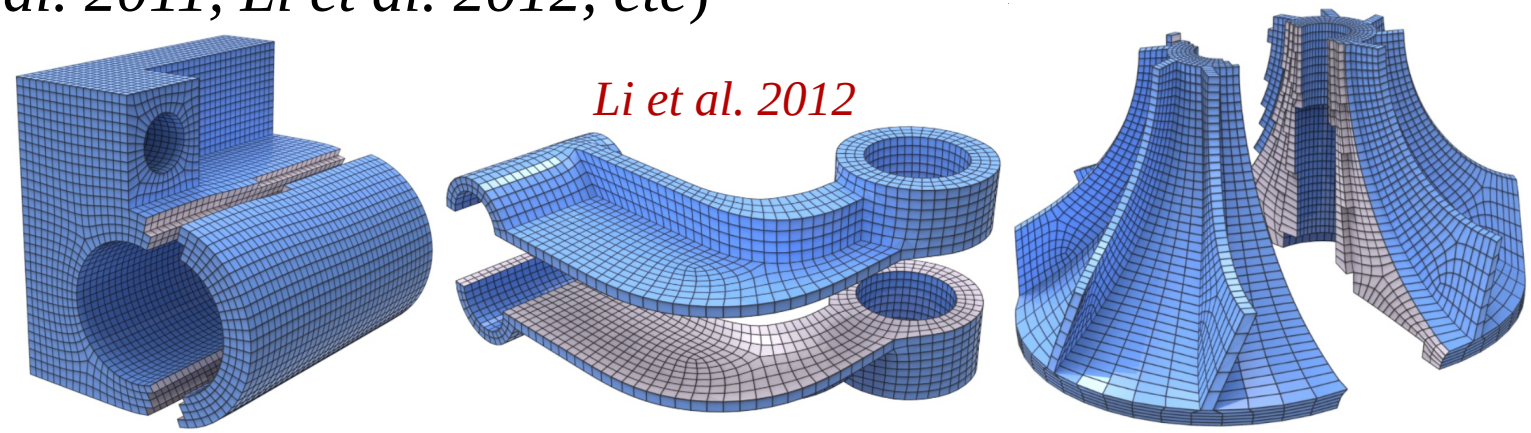
From boundary-aligned 3D frame field to full hex mesh

Hex-dominant meshes :

- frontal point insertion, tetrahedralization then recombination into hexahedra (*Baudouin et al. 2014*, *Bernard et al. 2016*, *etc*)
- polyhedral agglomeration (*Gao et al. 2017*)
- Parametric Global Parametrization (*Sokolov et al. 2016*)

Full hex meshes :

- CubeCover parametrization (mixed-integer problem) (*Nieser et al. 2011*, *Li et al. 2012*, *etc*)



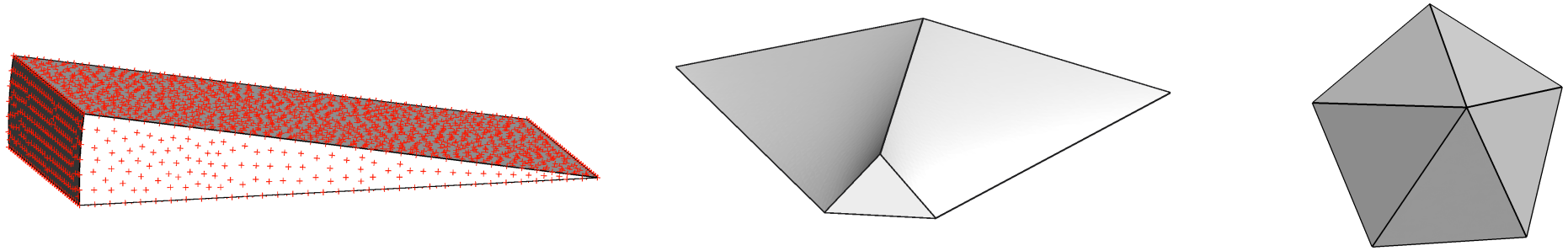
Li et al. 2012

Works for some models, but not robust due to frame field defects

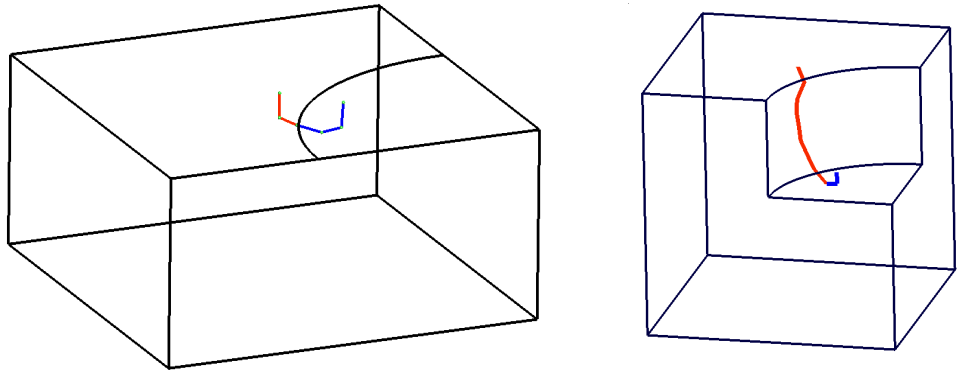
Current limitations of frame fields

Frame field based hex meshing is **promising** but **not robust** because :

1. Boundary conditions for frame fields can be non-trivial or impossible :



2. Frame field singularity graphs are not always "hex-meshable"



e.g. 3-5 singular curves

3. Frame field computation does not converge with non-uniform mesh refinement (gradients tend to infinity at singularities)

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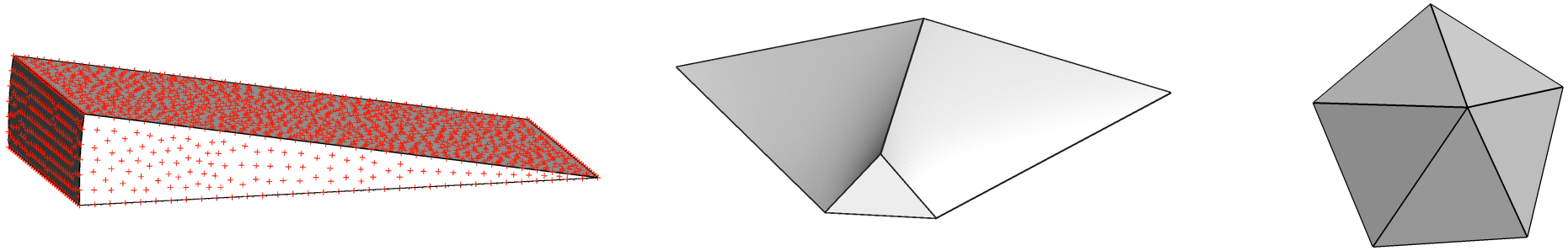

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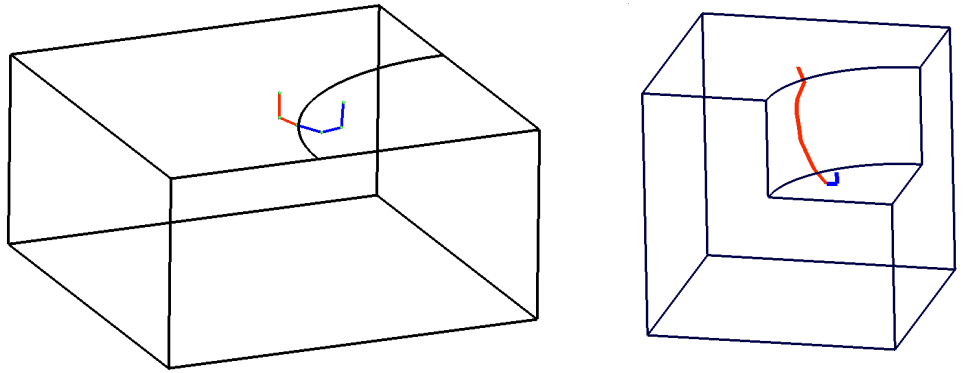


e.g. 3-5 singular curves

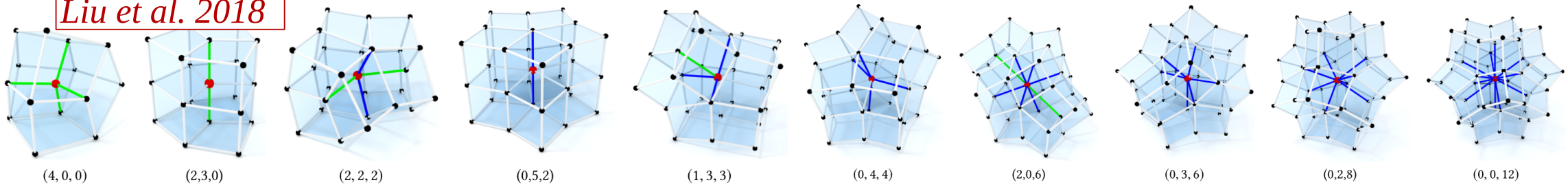
3. Frame field computation does not converge with non-uniform mesh refinement (gradients tend to infinity at singularities)

Frame field correction, introduction

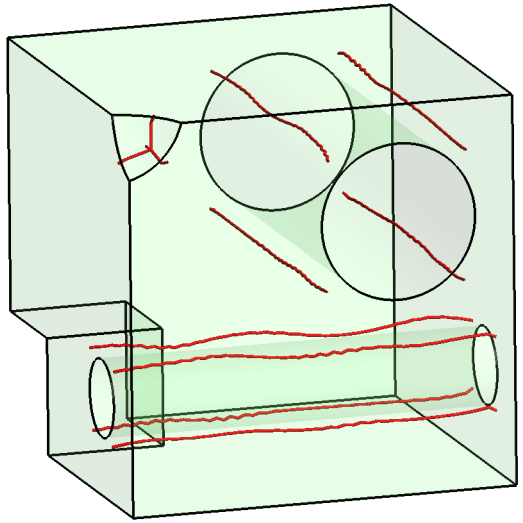
The singularity graph should have a valid hexahedral mesh topology

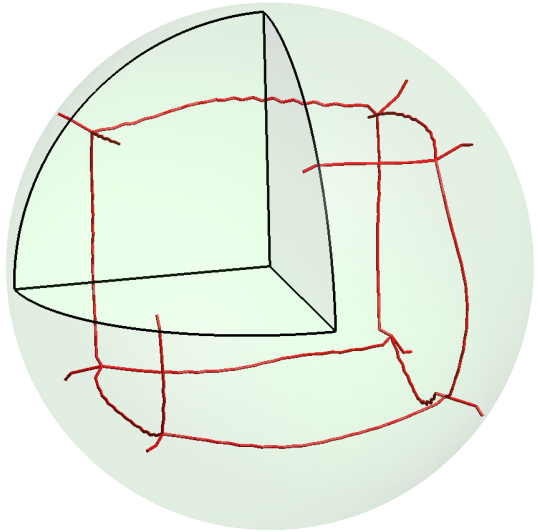
Valid vertex configurations in a hex mesh (correspond to triangulations of the sphere):

Liu et al. 2018



⇒ local necessary conditions on the validity of a singularity graph



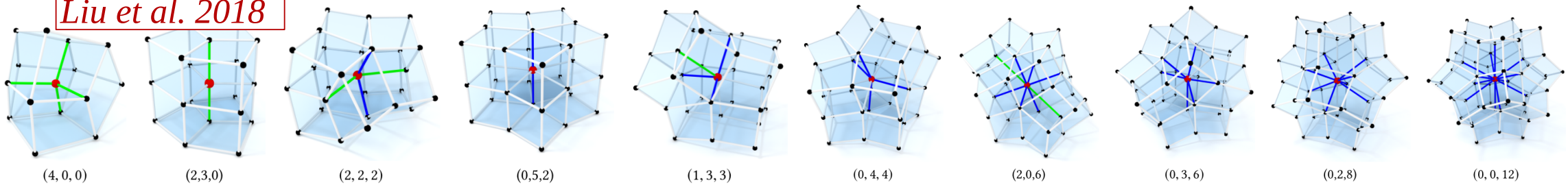


Global validity conditions is still an open problem

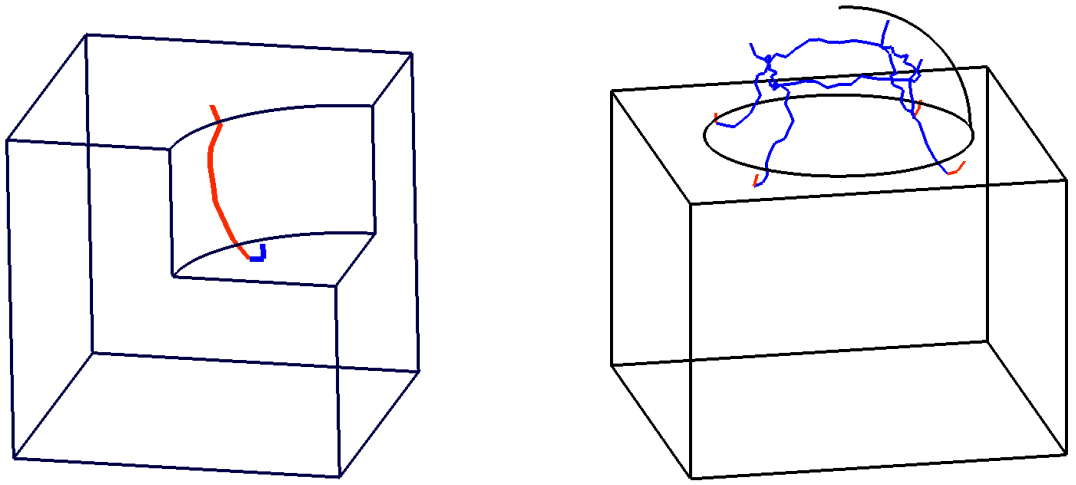
Frame field correction, introduction

Valid vertex configurations (restricted to edge valence 3, 4, 5):

Liu et al. 2018



In practice, state-of-the-art frame fields contain non "hex-meshable" singularities, e.g.:



References:

Ray and Sokolov 2015,

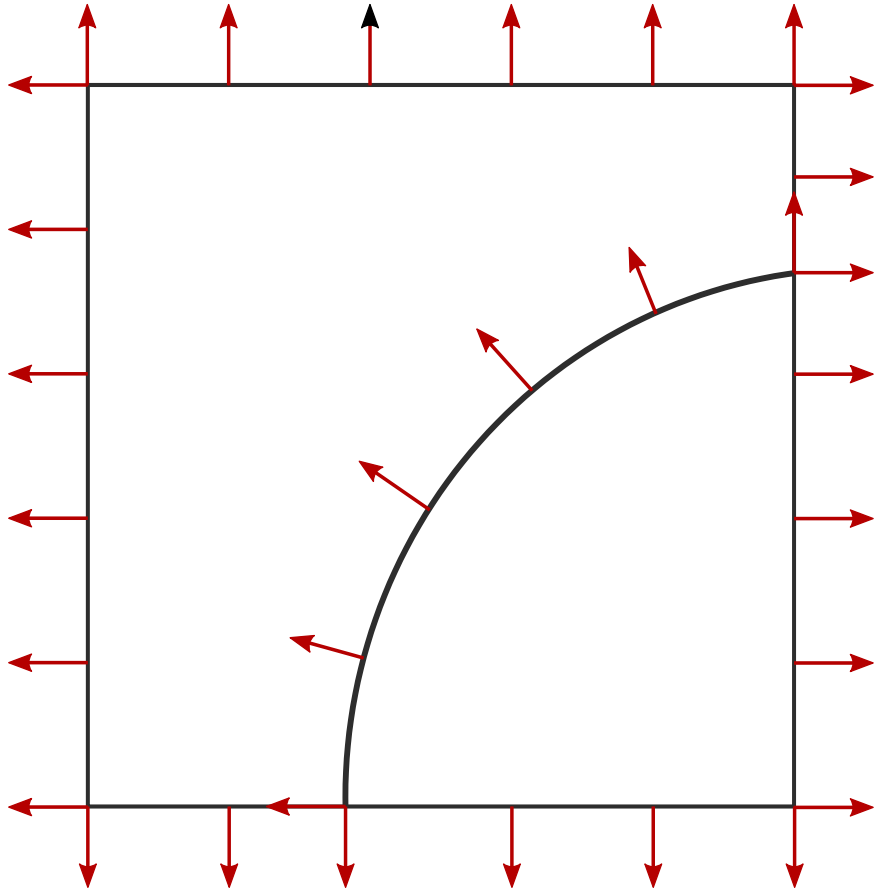
Viertel et al. 2016,

Liu et al. 2018

We focus on correcting **"3-5 singular curves"**

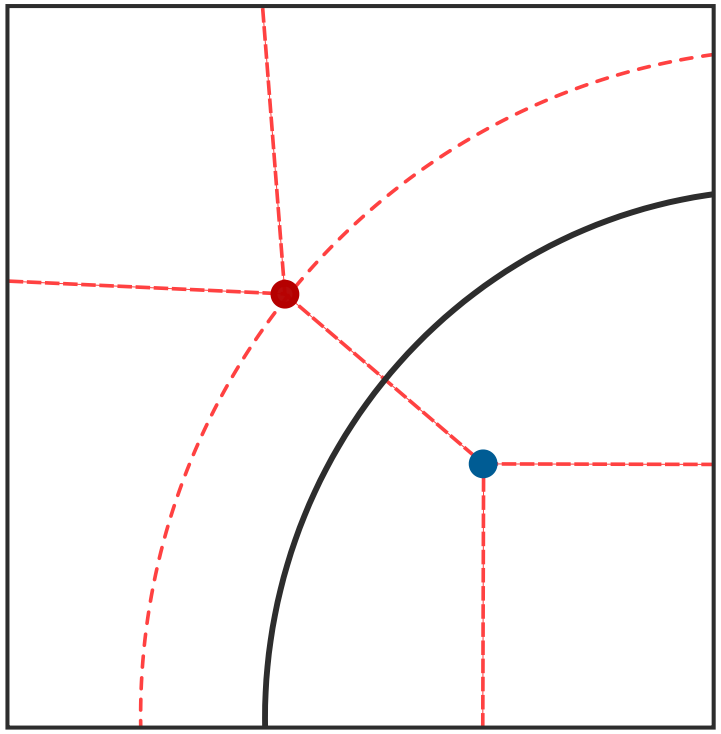
Feature curves in 2D: domain splitting

Square with imprinted arc



Boundary conditions (red arrows)

Quad decomposition (dashed lines)

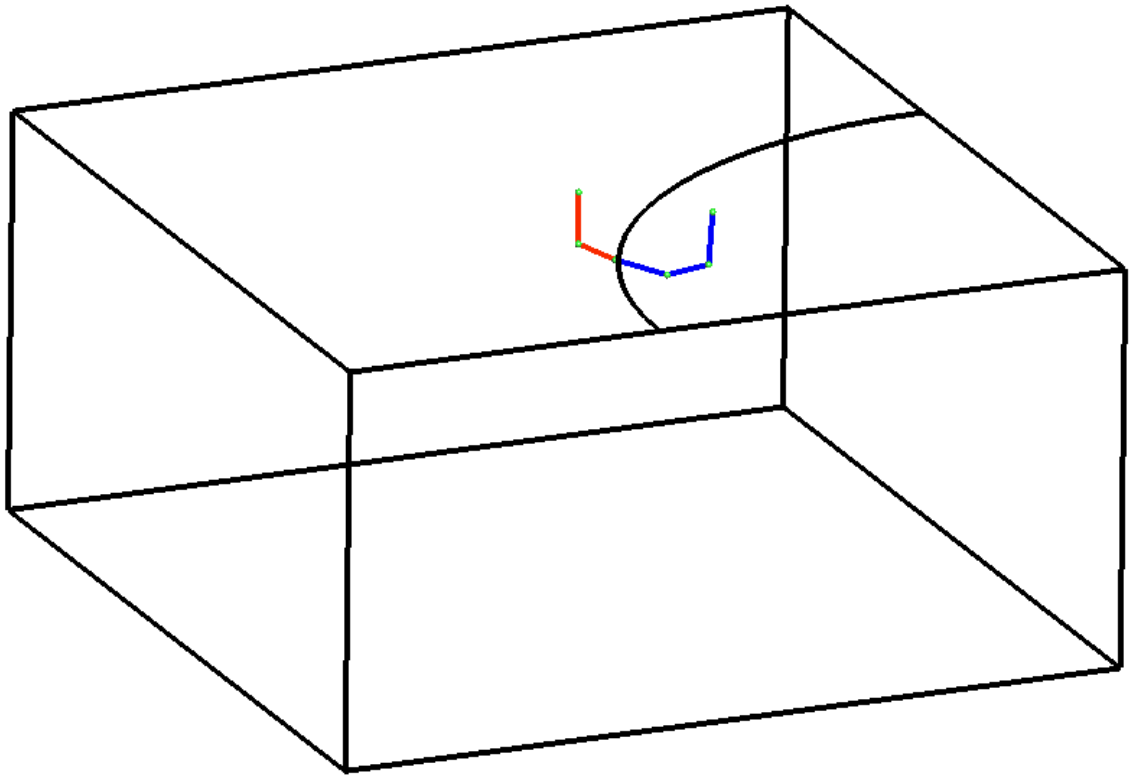


Cross field singularities :

- +1/4 (valence three)
- -1/4 (valence five)

Feature curves in 3D: only the boundary is split

Box with imprinted arc



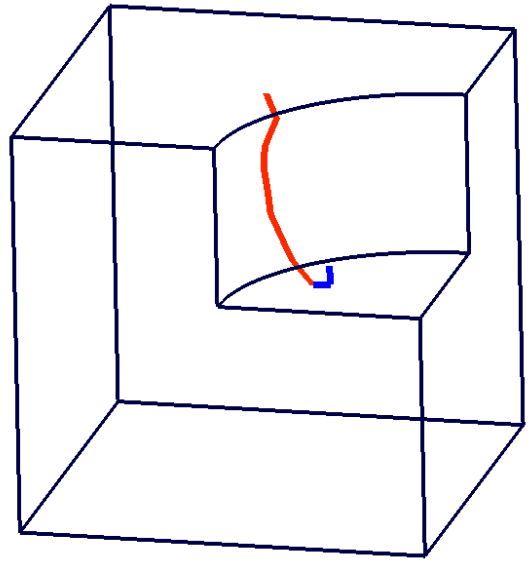
Non-meshable singularity graph:

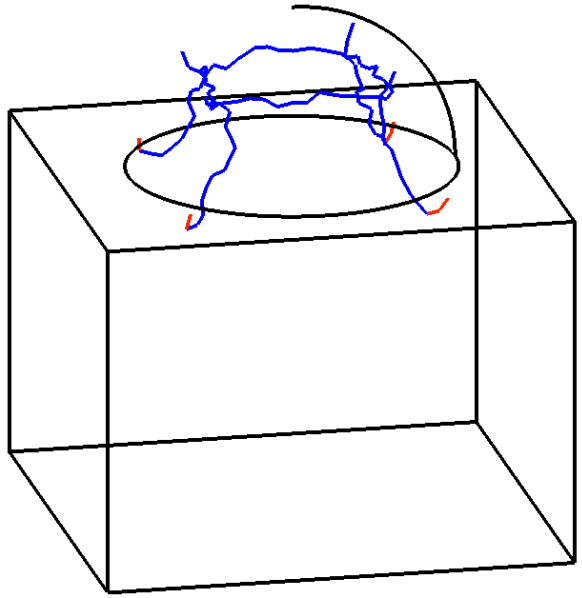
"3-5 singular curve"

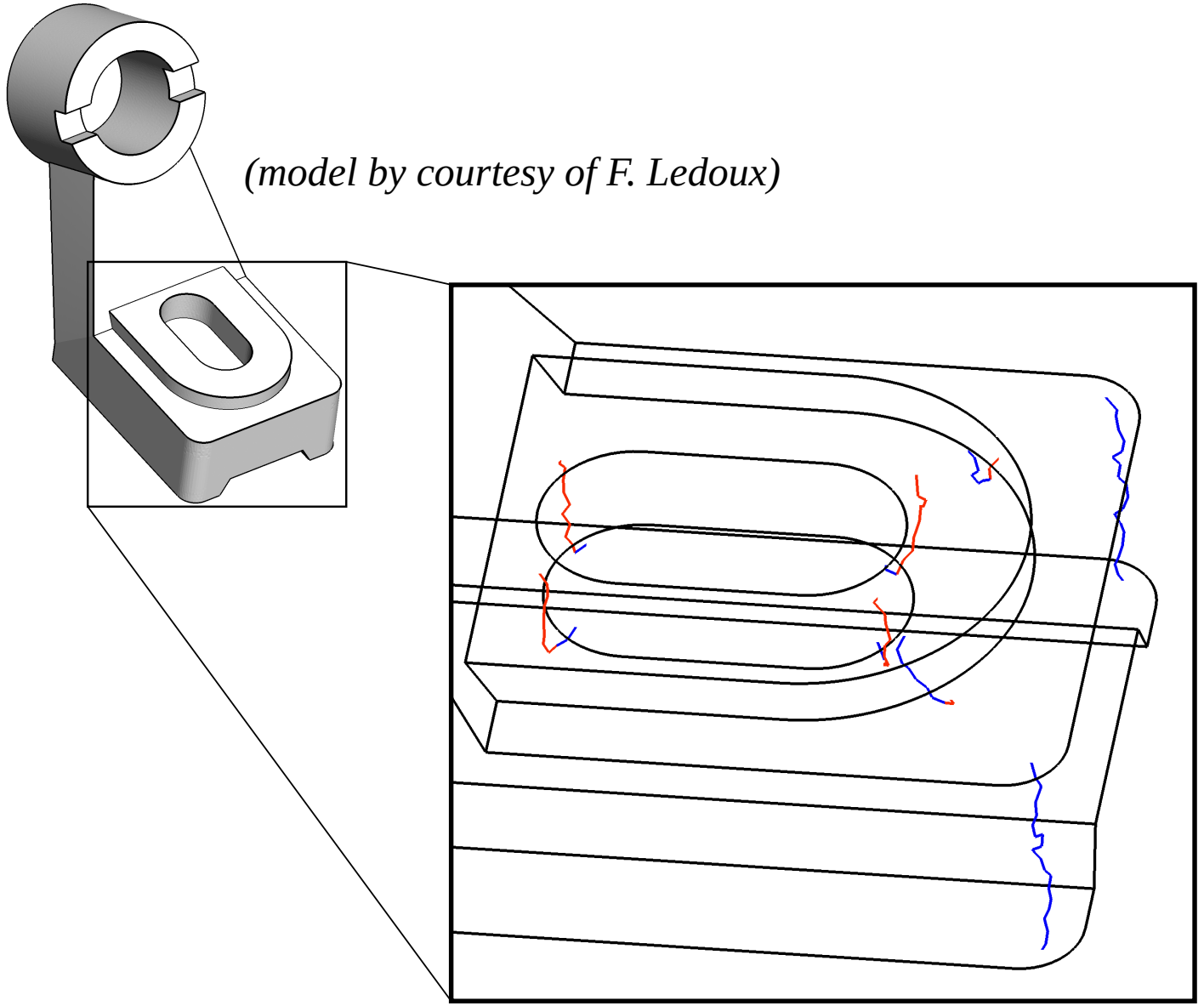
- minimize the Dirichlet energy (finite due to discretization)
- respects boundary conditions

Same for all existing frame-field solvers

Examples of non hex-meshable 3-5 singular curves

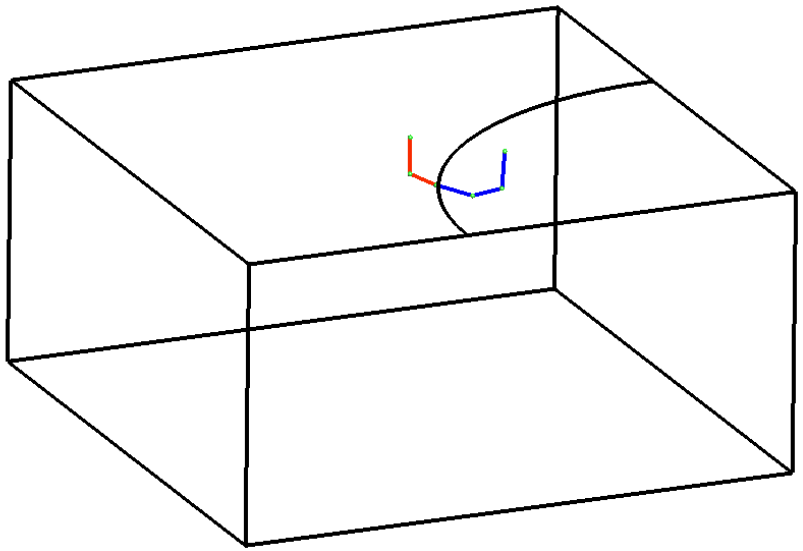


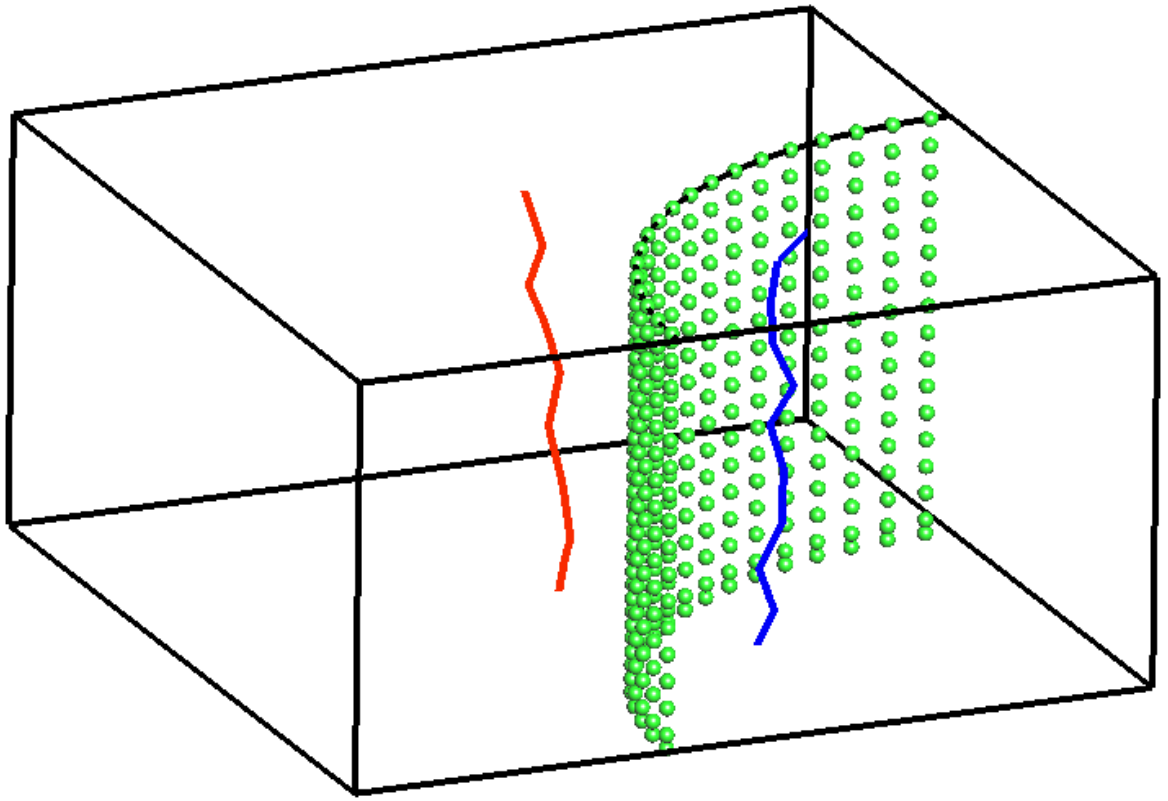




(model by courtesy of F. Ledoux)

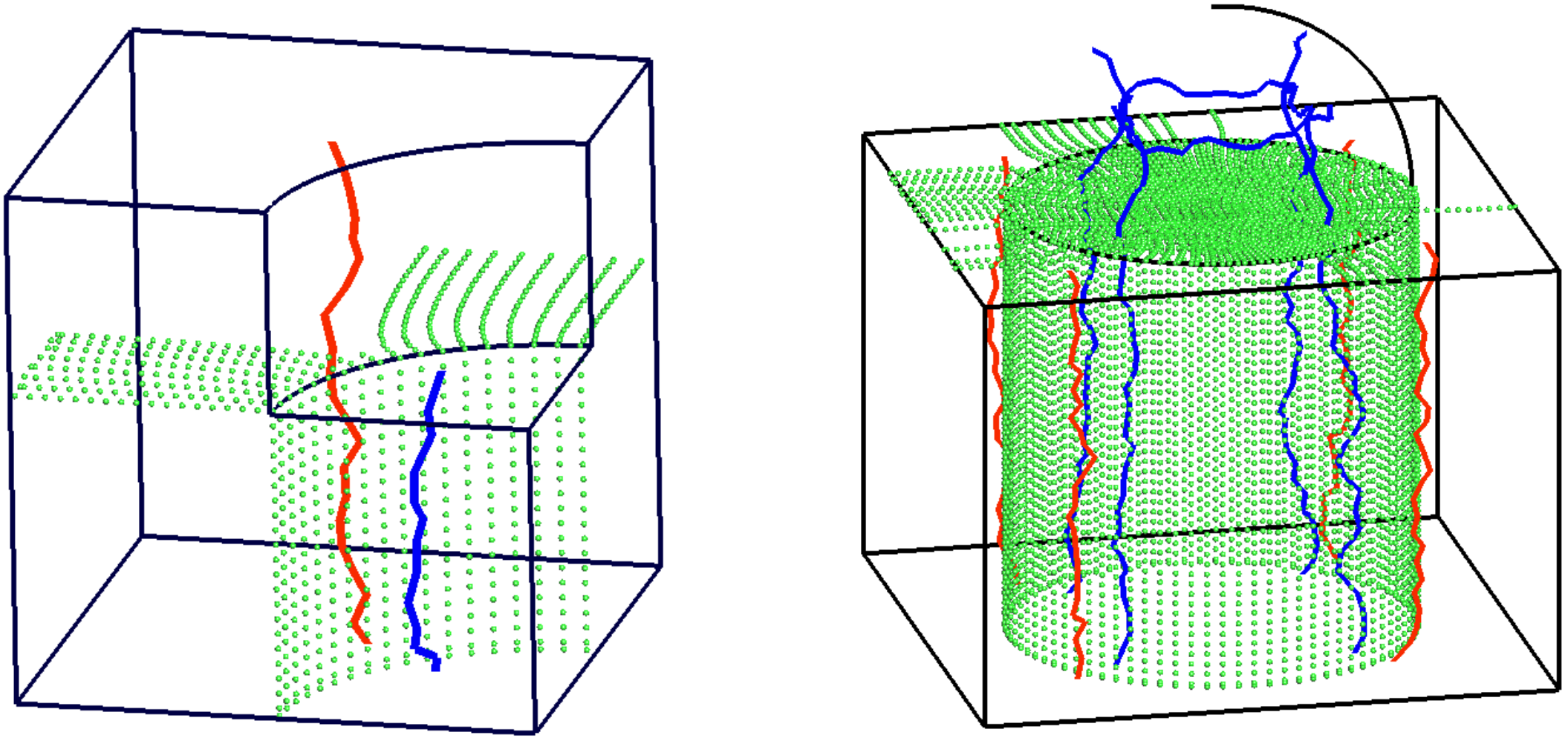
Frame field correction: extrusion of feature curve





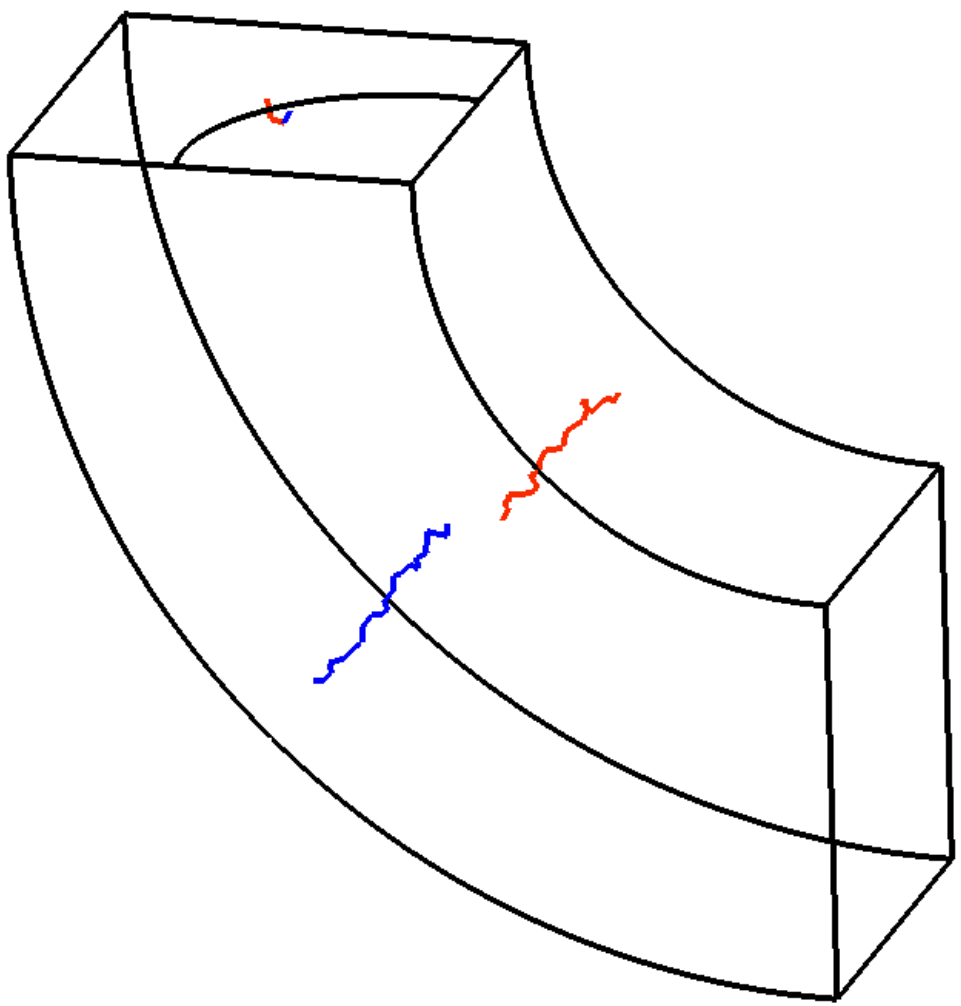
1. Trace streamlines from concave and curved feature curves (green)
2. Compute a new frame field with internal constraints
   (tangency to internal surfaces made of streamlines)

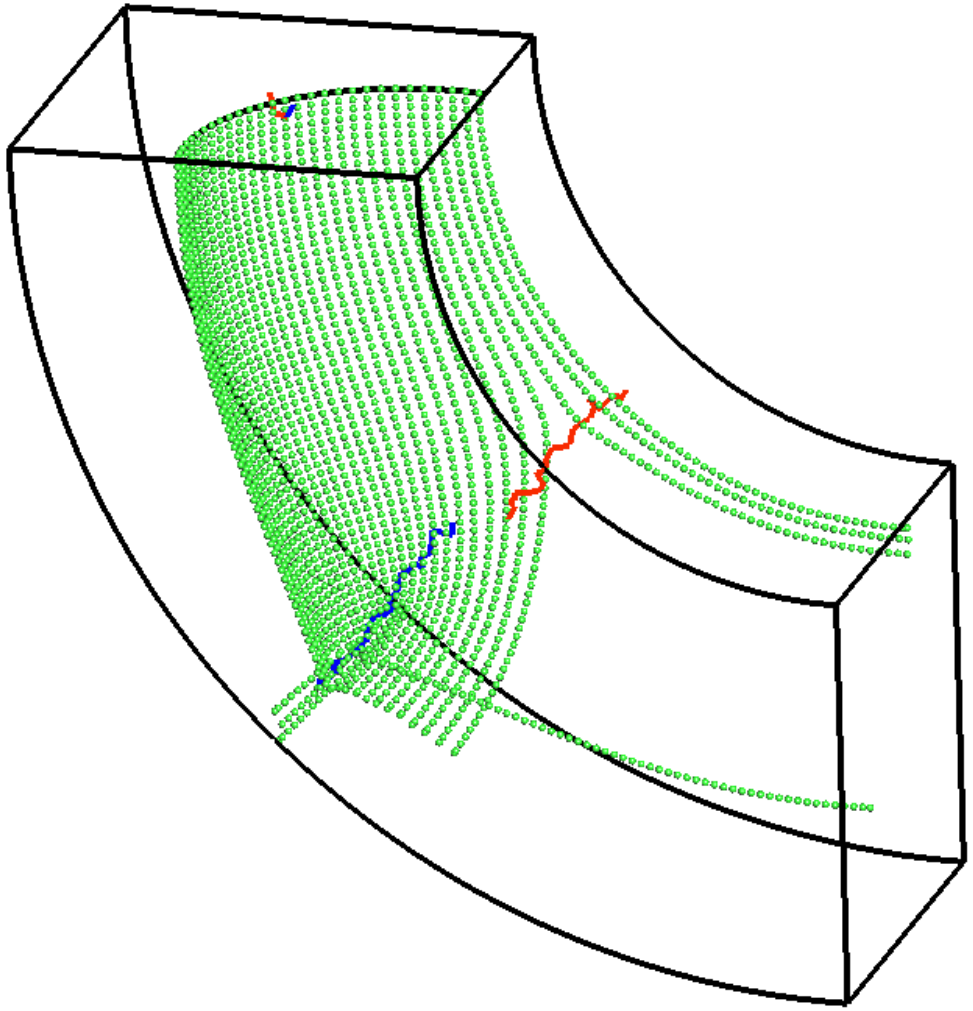
Frame field correction: extrusion of feature curve, examples



Internal constraints prevent the merge of val. 3 and val. 5 singularities

Frame field correction: extrusion of feature curve, failure case

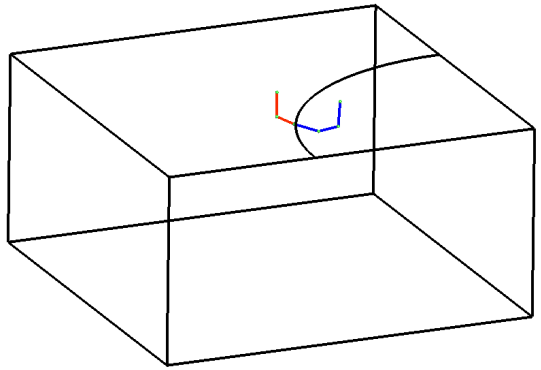


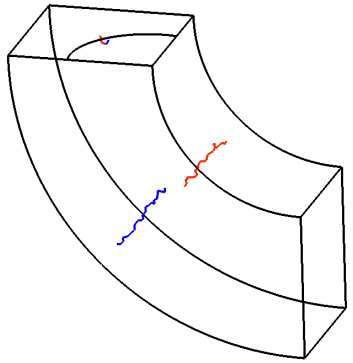


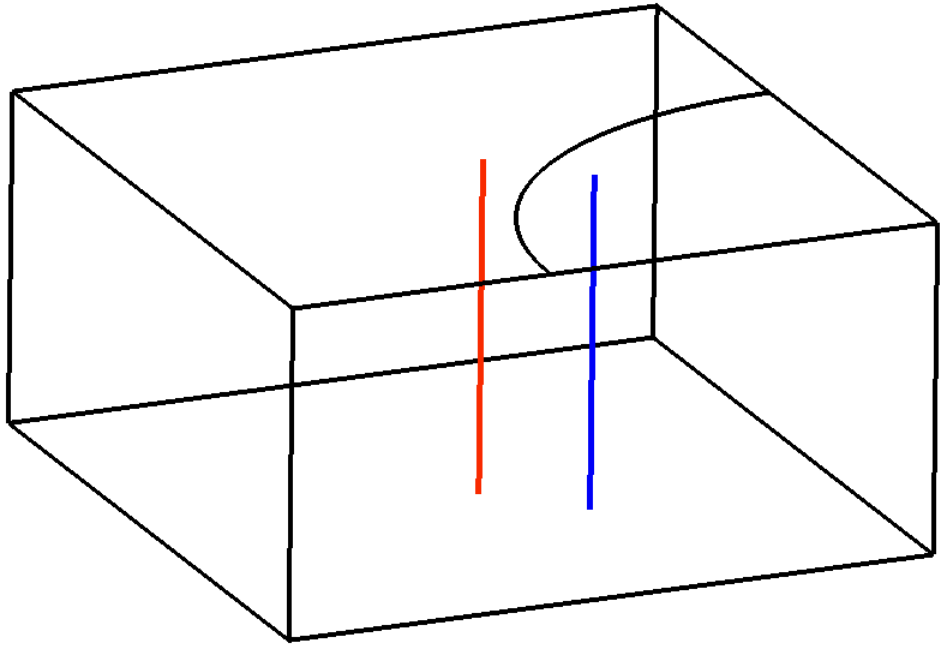
Interaction with other singularities,
extruded surfaces torn in multipe directions

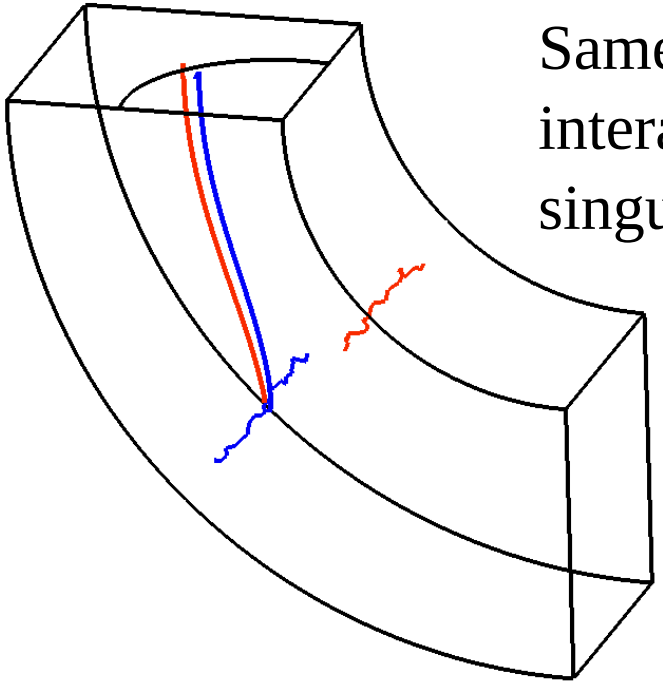
Frame field correction: extrusion of boundary singular nodes

Alternative approach: extrude singular nodes [Zheng et al. 18]



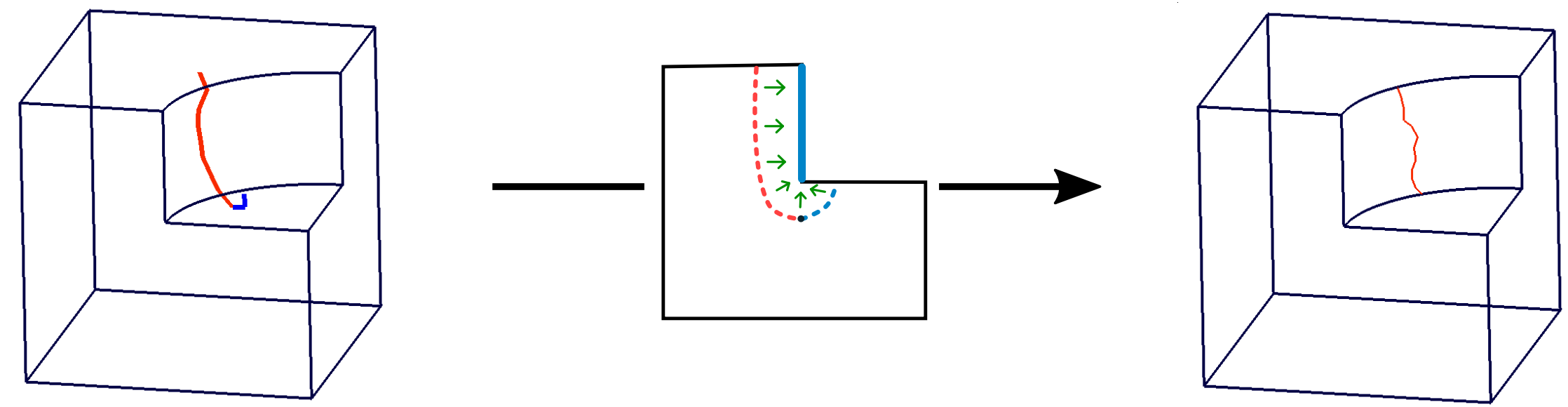






Same failure behavior: interaction with other singularities

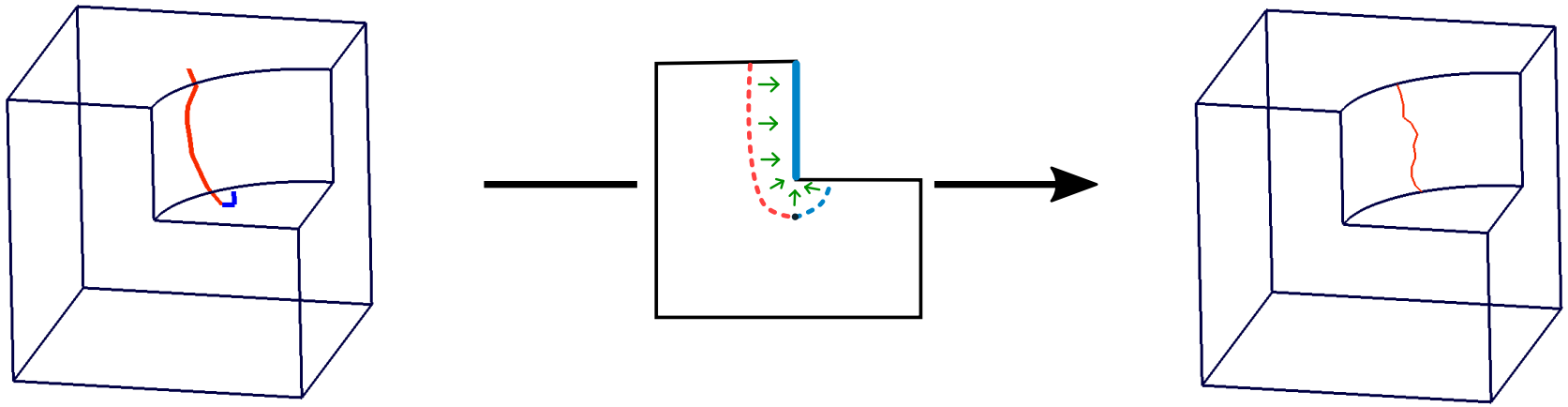
Boundary snapping of 3-5 singular curves



For each 3-5 singular curve:

- Move bdr. node extremity to closest feature curve
- Compute new boundary path between snapped extremities

Principle of 3-5 sing. curve boundary snapping



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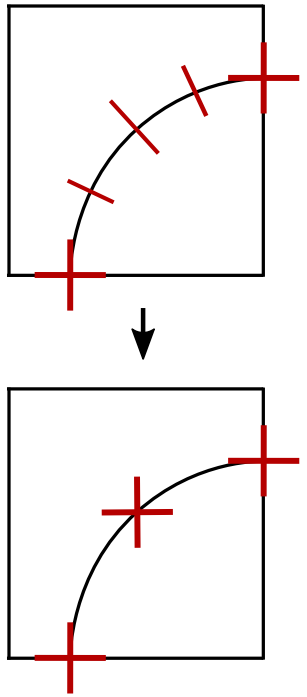
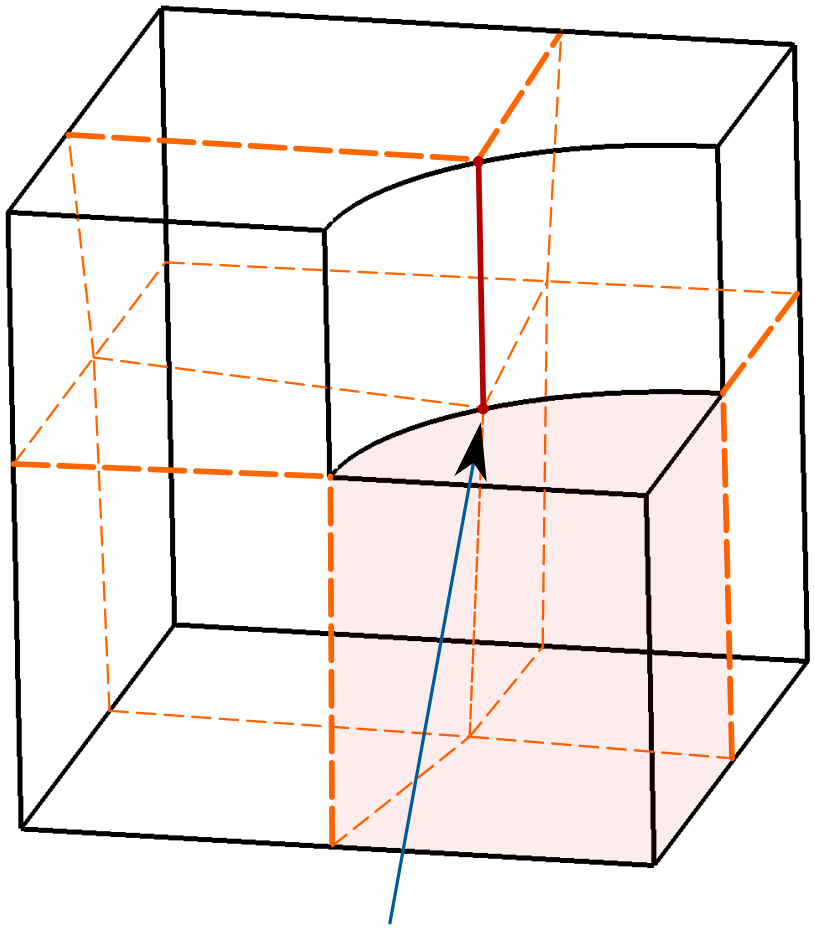


Change the frame field boundary conditions:

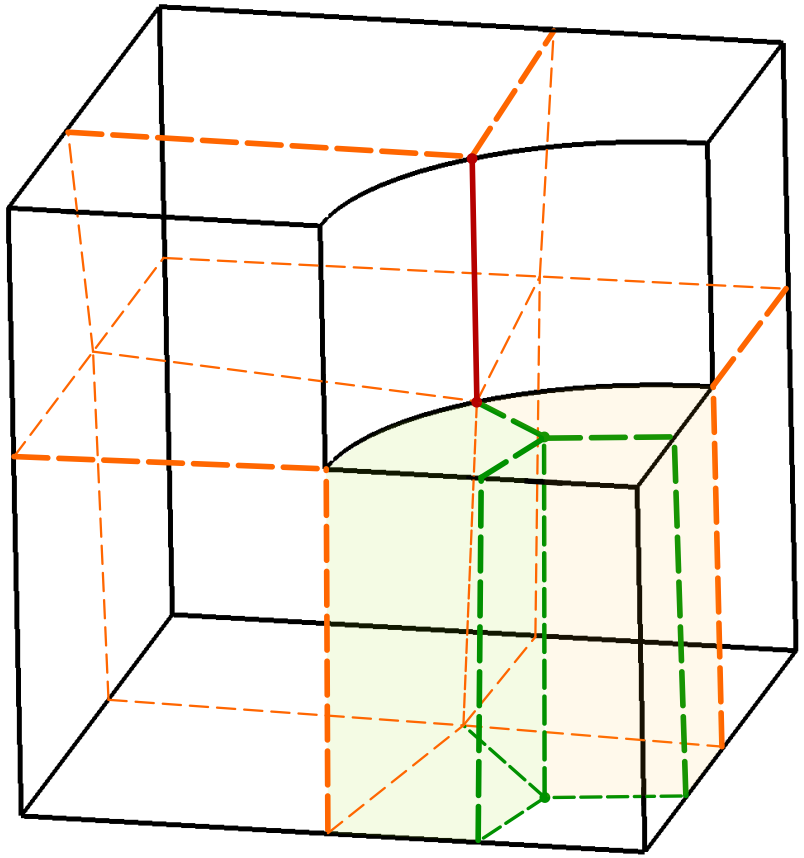
- On the snapped curves, imposed frames are rotated by 45° (around tangent)
- Close to snapped curves, alignement boundary conditions are removed

The new frame field is **topoligcally valid but no longer boundary-aligned**

Valid geometry with block decomposition refinement

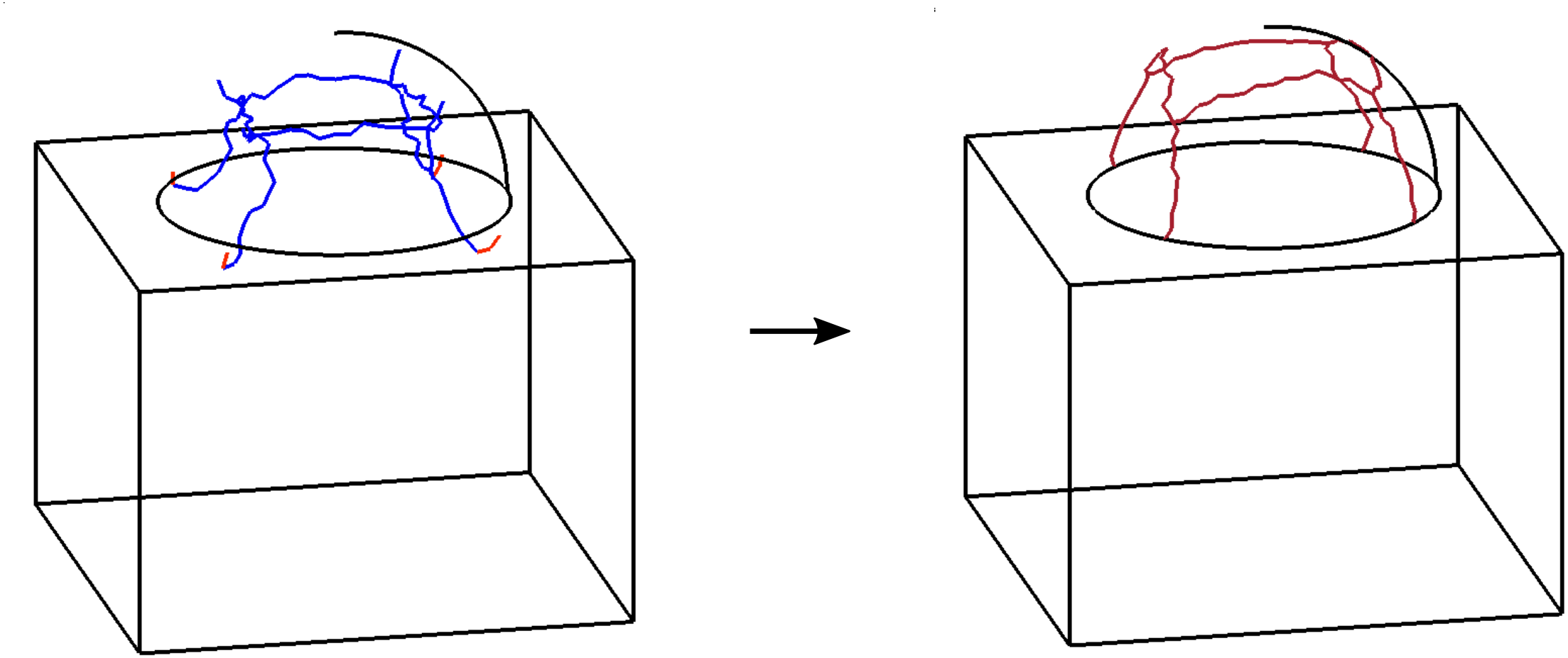


Corner with zero jacobian

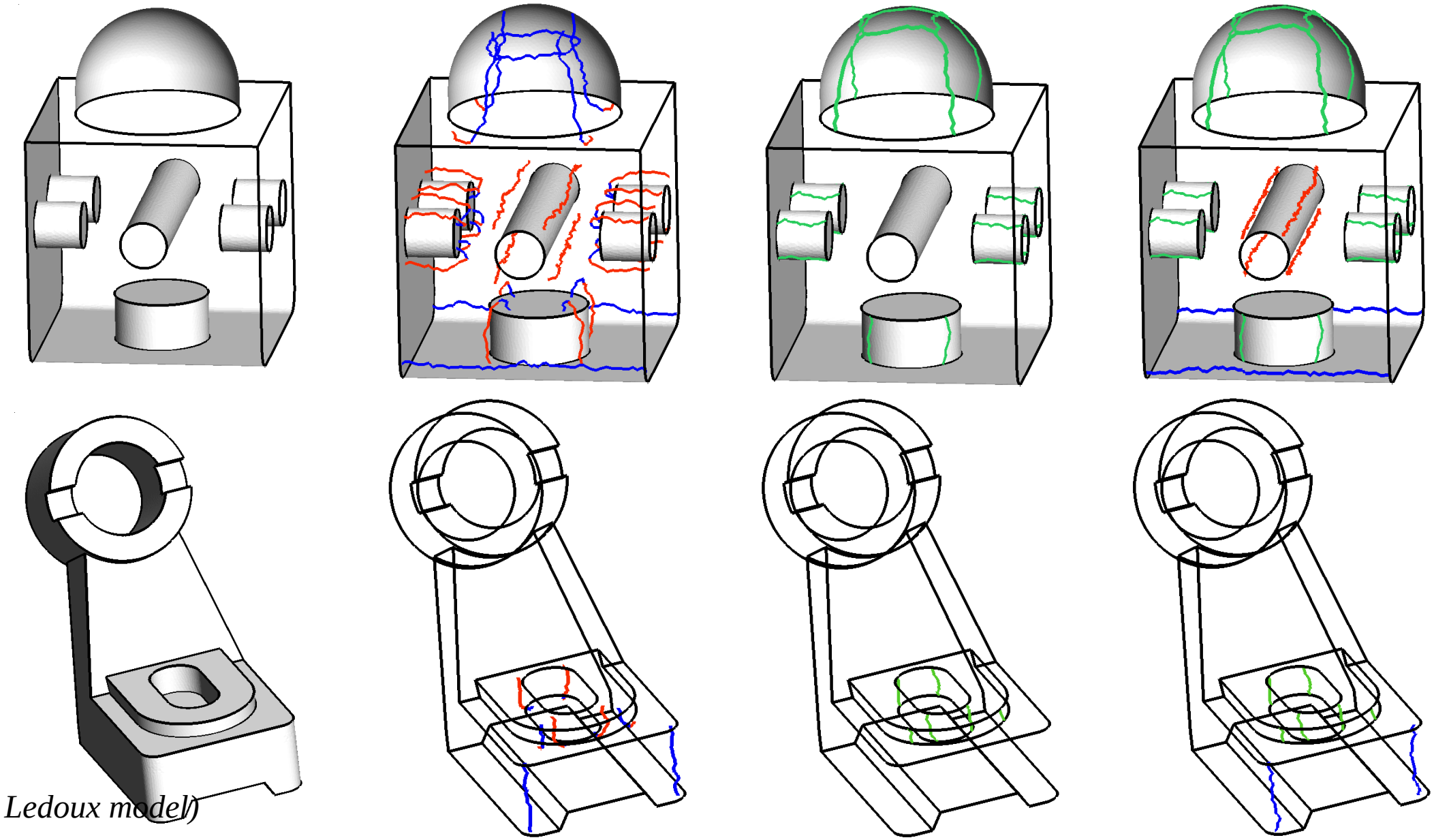


After refinement of the block, valid geometry

Frame field correction, results of 3-5 sing. curve boundary snapping



Frame field correction, results of 3-5 sing. curve boundary snapping

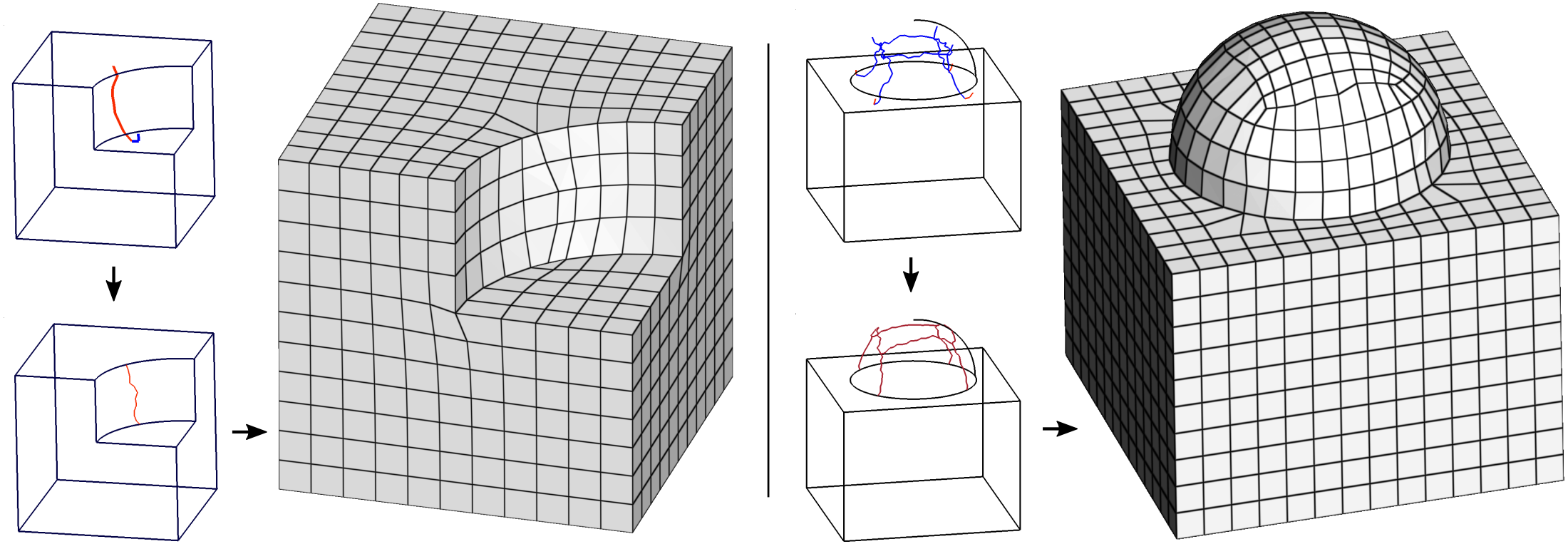


(F. Ledoux model)

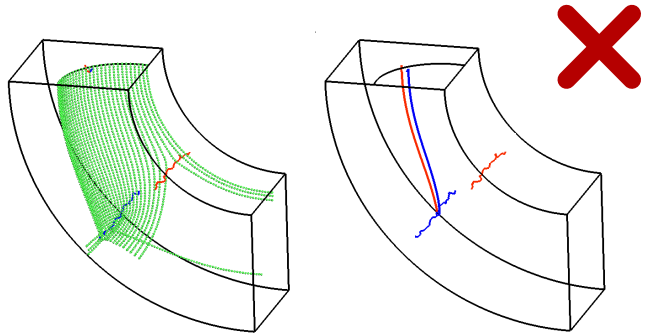
Frame field correction, results of 3-5 sing. curve boundary snapping

To get hexahedral meshes :

- Frame field with new BCs (changed after snapping)
- CubeCover parameterisation (using CoMISo [Bommes et al. 2011])
- Hexahedra extraction (using HexEx [Lyon et al. 2016])

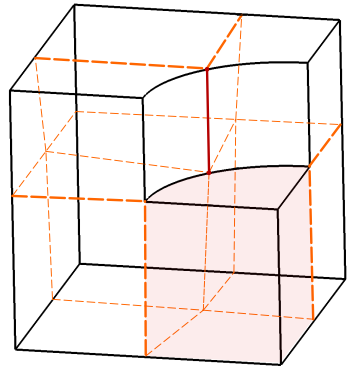


Conclusion on frame field correction



Extrusion (curve or bdr. sing.) not reliable

attempt of global correction



3-5 singularity snapping : topologically valid frame field

local correction by removing singular curves

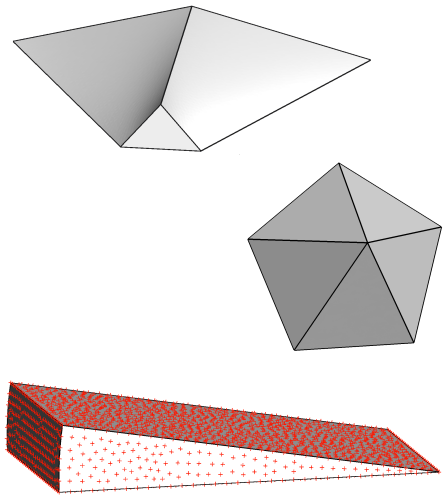
no longer boundary aligned

require block refinement

limited to 3-5 singular curves close to the boundary

Conclusion

Frame fields require more work before robust full hex meshing
(consistent boundary conditions, high resolution, graph validity)
but very promising (contain the block decomposition information !).



Thank you for you attention

Any questions ?